Electrolytic Tank Analogue Design and Application of Automatic Control

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Electrolytic Tank Analogue
Design and Application of Automatic Control

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Synopsis

It is well known that electrolytic tank analogue, which is used to measure the electric potential and the potential gradient of the electrostatic field formed analogically by the model electrodes in an electrolytic tank by a submerged probe tip, is a usual method not only for the analysis of electrostatic equipotential line distribution, but also for the solution of other fields where Laplacian equation is followed, for instance, stress analysis in the field of material, hydraulics and thermal conduction problem by analogical consideration. Furthermore, by an adequate arrangement of mechanisms, a design of electric network function, and a plotting of electron trajectory in an electrostatic field are made possible. Such wide usefulness of electrolytic tank analogue in practical purpose has resulted from the simple construction of mechanism, and the easy operation of equipment. From the viewpoint of practical use, however, many problems to be solved concerning electrolytic tank analogue still remain. This means that higher accuracy and much easier operation are necessary.

In considering the accuracy of electrolytic tank analogue, the error of measurement can be said to depend principally on the following factors: the polarization of liquid near the electrodes, the perturbation of an electric field by a submerged probe, the surface tension, and the mechanical construction. Other workers reported that the factors to be concerned about the accuracy are the same as those described above, but few have made reports on the quantitative measurement of individual factors.

On the other hand, in spite of the usefulness of the electrolytic tank, the manual control is tediousness as well as the waste of time to the operator. Then the application of automatic control has been desired. For this reason, it has been often tried to apply servomechanism for the electrolytic tank analogue, and some investigators have reported an automatic electrolytic tank analogue, especially, the automatic equipotential line plotter, the automatic electron trajectory plotter, and the network function designer. However, many problems concerning the mechanism and operation of these automatic equipments still remain to be solved for practical purposes.

We tried an investigation for the practicalization of electrolytic tank analogue, especially the measurement of accuracy, and the automation of the equipments. First, the measurement of accuracy about the probe impedance and the polarization of liquid were tried, and then an automatic equipotential line plotter and an automatic electron trajectory tracer were constructed. This paper gives the summary of the work on the electrolytic tank analogue.

In Chapter I, it is described the problems about the electrolytic tank analogue and a general explanation of the present authors' work as well as a review of previous works by others.

In Chapter II, the mechanism and the general theory of electrolytic tank analogue is described. The measurement of accuracy is also given on the basis of the experiment of the authors. With respect to the polarization of liquid, the excess resistance and the capacity caused by the polarization were measured in many kinds of combinations of liquids and electrode metals. On the other hand, in reference to the perturbation by the probe, the changes in probe impedance depending on the submerged depth in every probe metals were measured from the viewpoint of minimizing the field perturbation and getting the adequate input impedance of the first stage vacuum tube amplifier of a null detector*.

* The details of experiment well be reported in the other paper.

In Chepter III, it is reported in detail the automatic equipotential line plotter which was constructed on the principle of applying the automatic control to the electroyltic tank analogue. This equipment is an extension of the usual electrolytic tank method to automatic plotting by means of servomechanism; it causes the probe to keep itself on the equipotential line to be plotted, compensating any deviation, as it travells in one direction.

The mechanism of the equipment and several examples of the equipotential line maps are given.

In Chapter IV, it is described the automatic electron trajectory tracer in a electrostatic field. The method of measuring the electron trajectory in two demensional electrostatic fields is based on the fact that the radius of curvature of electron trajectory is equal to the ratio $2V/\varepsilon_n$ when a pair of probe wires dipped into the electrolytic tank detects V (the potential of the field formed by model electrodes where electron exists) and ε_n (the potential gradient normal to the direction of motion). The plotting is achieved by the continuous adjustment of the radius of curvature of the probe path, compensating the deviation as the probe moves on the surface of electrolyte. Mechanical design, electrical curcuit, and the performance are also discussed.

In Chapter V, it is found the summary of the work about the general accuracy of electrolytic tank analogue and the attempt to apply the automatic control to it.

Finally, in Appendix, it is described a new type A.C. potentiometer by which amplitude and phase difference of A.C. voltage can be measured. The characteristics of this apparatus is based on the principle that the amplitude and the phase difference of unknown A.C. voltage are measured by comparing with the standard voltage having the same frequency.

In this case, the standard voltage is converted to the form a_0+jb_0 by a quadrature generator. This is a useful instrument for detecting very rapidly the resistance of electrolyte or the probe impedance. It is also a convenient tool for comparing two A.C. voltages as in the case of designing servomechanism.

I. Introduction

1. General explanation

It is well known that when physical phenomena or engineering problems are considered, they can be solved by measuring analogically phenomena different from the above but in the same mathematical relations with them. Similarly, some mathematical equations can be solved by transforming the numerical figures into physical quantities by means of adequate mechanical or electrical equipments. Such equipments, called analogue, have come to play more important role in the analysis of physical and engineering problems with the recent development of electronic computer. The typical illustration of analogical method can be seen in the analogue computer. In many kinds of analogues, the electrical analogue by which physical quantities can be transformed their numerical values into electrical quantities, is most useful because of simple construction and easy operation. The basic principle of electrical analogue lies in the analogical formation of the physical phenomena in electric field, which can be classified into the following three kinds from the viewpoint of medium in which the electric field is formed(1):

(1) Conducting paper analogue,

(2) Electrolytic tank analogue,

(3) Circuit element analogue,

(1) G. Liebmann, British J. Appl. Phys., **4** (1953), 193.

The last method is useful for high degree of accurate measurments and is often used as an analogue computer. But the system is to be much complicated[(2)]. On the other hand, the second method allows simple mechanism and easy operation. Therefore, this method is used in many fields of engineering problems, such as in the fields where Laplacian equation holds, for example, stress analysis of material, hydraulic flow, and thermal conduction, in addition to the electronic engineering problems. This method, however, leaves many questions to be solved .about the accuracy. First, the polarization of electrolyte occurs near the electrodes, and a submerged probe produces the perturbation in the field of electrolyte. In the works of other authors, they gave their attentions to these factors, but from practical purposes, sufficient details cannot be found.

Furthermore, since its manual operation is tedious and waste of time to the operator, an application of automatic control is desirous. Applications of automatic control to the electrolytic tank analogue are reported by some investigators to get rid of inconviniences from experiment. From practical purposes, however, this problem leave also many questions to be solved.

For the reason described above, the authors have investigated the accuracy of measurement and the application of servomechanism to electrolytic tank analogue. This paper gives a summary of the works on the design of electrolytic tank analogue and the application of automatic control paticularly to the equipotential line plotter and the electron trajectory tracer.

2. Review of previous works

The paper of electrical analogue was first published in 1845 by G. Kirchhoff[(3)], who tried the experimental electric field plotting by a thin metal film instead of the current conducting paper. In spite of the simplicity of mechanism, this technique was not applied for the next 100 years, because of the lack of uniformity of resistance of the metal film and the difficulty of contact. Recently, it was reported by several authors[(4)] that the current conducting paper can be used for forming an analogical electric field. The electrode is formed by painting on a sheet of paper with the conductor, which is directly supplied with voltage. This method produces no polarization near the electrode, and its operation is very easy. The resistance of such paper is about 1000~5000 Ω/cm, and the accuracy of conducting paper analogue was reported to be about 2 per cent. The question about the conducting paper analogue is that the resistance of paper is not quite isotropic, or the ratio of specific resistance in the two principal directions perpendicular to each other is not equal to unity. Furthermore, difficulty still exists in the occurrence of noise at the contact of probe.

The use of the electrolytic tank analogue was first reported by Adams[(5)] in

(2) G. Kron, Elect Engng, N. Y., **67** (1948), 672.
(3) G. Kirchhoff, Ann. Phys., Lpz., **64** (1845), 497.
(4) C. T. Murray, and D. L. Hollway, J. Appl. Phys., **24** (1953), 110.
(5) W. C. Adams, Proc. Royal Soc., **23** (1875), 280.

1875. The principle of this electrolytic tank analogue is based on the fact that when the model electrodes are supplied with voltage, equipotential line of field formed by these electrodes can be plotted by dipped probe. Furthermore, Bowman and Nicoll(6) found a method which depends on the fact that the section of Faraday tube of three demensional field with axial symmetry forms sector type and such field can be analyzed by the wedged tank. By the use of analogy between the electric field and other fields, analysis of many kinds of field has been tried. First, stress of material was analyzed by the analogical form with elastic fields(7). Disign of electric network function was also tried by Boothroyd(8) and others,(9),(10),(11),(12) in analogical consideration between the electric field and higher order porinomial. Thermal conduction was also considered by the electrolytic tank(13).

D. Gabor(14) introduced a method of plotting the electron trajectory in an electrostatic field by the electrolytic tank, based on the fact that the radius of curvature of electron path is twice the ratio of electric potential and potential gradient at every position of an electron to be plotted. The electrolytic tank analogue was also extended to the design of electric power machine(15) and electron lens(16). At the same time, the accuracy of electrolytic tank analogue was considered in many respects, for example, the perturbation of field by the insertion of a probe, the polarization of liquid, the effect of surface tension and the mechanical error.(17),(18),(19). Thus, the electrolytic tank analogue is found to be convenient in measuring the field where Laplacian equation holds.

The manual operation of electrolytic tank analogue is, however, tedious, and therefore the application of automatic control is necessary. In the same year that D. Gabor introduced the method of measuring electron trajectory by electrolytic tank analogue, D. Langmuir suggested the possibility of the application of automatic control to Gabor's equipment(20) which controls the radius of curvature of electron trajectory by the null-bridge method. This automatic electron trajectory plotter was reported in detail in 1950(21), and developed dy Baker(22). Another measuring

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(6) M. Bowman and H. Nicoll, Nature (London), **39** (1938), 140.
(7) M. Hetenyi, *Hand book of experimental Stress analysis.* John Wiley and Sons Inc. N.Y. (1950), 737.
(8) A. R. Boothroyd, E. C. Cherry and R. Maker, Proc. I. E. E., **96** part 1 (1949), 163.
(9) W. W. Hansen and O. C. Lundstrom, Proc. I.R.E. **33** (1945), 528.
(10) R. E. Scott, Proc. I.R.E., **40** (1952), 970.
(11) S. Darlington, B.S.T.J., **30** (1951), 315.
(12) W. H. Huggins, Proc. I.R.E., **36** (1948), 421.
(13) F. Reiniger, Philips Technical Review, **18** (1956), 52.
(14) D. Gabor, Nature (London), **139** (1937), 373.
(15) D. Macdonald, Proc. I.R.E., **100** part 2 (1953), 145.
(16) V. K. Zuorikin, G. A. Morton, E. G. Ramberg, J. Hillier and A. W. Vance, *Electron Optics and Electron Microscope*, John Wiley Co. (1945).
(17) P. A. Einstain, British J. Appl. Phys., **2** (1951), 49.
(18) K. F. Sander and J.G. Yates, Proc. I.E.E,. **101** part 2 (1954), 167.
(19) P. A. Kenedy and G. Kent, R.S.I., **27** (1956), 916.
(20) D. R. Langmuir, Nature (London), **139** (1937), 373.
(21) D. R. Langmuir, R.C.A. Rev., **11** (1950), 143.
(22) B. O. Baker, British J. Appl. Phys. **5**. (1954), 191.

method was suggested by Oatley, Sander and Yate[23], by integrating the motional equation of electron with a mechanical integrator.

In reference to the equipotential line plotter, P. E. Green Jr.[24] tried the application of servomechanism. The usual bridge methode was extended to the automatic control. In this case, the deviation between the probe voltage and the reference voltage in any direction is compensated by servomechanism during the travel of the probe at a constant speed in a direction perpendicular to it. This, however, makes impossible to trace the contour type of equipotential line. Mickelson[25],[26],[27] tried the another method to adoid this difficulty.

Rrecently other uses of electrolytic tank analogue were reported; for instance, the plotting of electrostatic flux[28], and the plotting of electron trajectory in the field where both electrostatic field and magnetic field exist[29]. The electrolytic tank analogue is used in many fields, because of easy operation and simple construction. However, it leaves many questions on the accuracy and the mechanism of automatic control[30],[31].

3. Statement of the problem

As described in the last paragraph, the use of the electrolytic tank analogue makes possible the measurement of electric potential and potential gradient and it is also possible to measure the field of thermal conduction, hydraurics, etc, where Laplacian equation follows. Furthermore, the electron trajectory can be plotted by the adequate combination of the measuring methods. The accuracy can be considered to depend principally on the following terms[32].

(1) Polarization of electrolyte near the electrodes,
(2) Perturbation of field, produced by a submerged probe tip,
(3) Effect of surface tension,
(4) Mechanical error.

According to the previous works, the polarization of electrolyte suddenly decreases when the supplied voltage of model electrodes becomes higher than 1000 cycle in frequency. The investigation of polarization from the viewpoint of physics and chemistry was done by Kohlrausch[33], Jones[34], and Jaffé[35]. The combinations of

(23) K. F. Sander, C. W. Oatley and J. G. Yate, Proc. I.E.E., part 3, No. 6099 (1952), 169.
(24) P. E. Green Jr., R.S.I., **19** (1948), 646.
(25) J. K. Mickelson,G .E. Rev., **25** (1949), No. 11.
(26) T. Isobe, Electronician (Japan), **2** (1953), 47.
(27) R. Gelfand, B. J. Shinn and F. B. Tuteur, *Electronics and Comunication* (1955), 73.
(28) L. Beaver, J. Appl. Phys., **28** (1957), 579.
(29) D. Hollway, Proc. I.R.E., part **B, 103** (1956), 155.
(30) Y. Tanabe and S. Yamada, Bulletin of the Research Institute for Scientific Measurement, Tohoku Univ.. **4** (1955), 99.
(31) Y. Tanabe, S. Yamada and T. Suzuki, Bulletin of the Research Institute for Scientific Measurements, Tohoku Univ., **5** (1957), 77.
(32) (17), (18) cf.
(33) F. Kohlrausch, Wied. Ann., **49** (1893), 249.
(34) G. Jones and D. M. Bollinger, J. Amer. Chem. Soc., **57** (1935), 272.
(35) G. Jaffé and H.C. Chang., J. Amer. Chem. Soc., **20** (1952), 1071.

electrolyte and electrode material which produce small polarization, should be considered not only from the enginering view point but also from the economical view point. It is expected that this effect should independent on the impurity and the temperature of the electrolyte, since a large quantity of electrolyte is required. Although the previous works reported the accuracy of tank analogue, the quantitative check was not found. The perturbation of field occurs by the insertion of a probe in the electrolyte. It is well kown that this effect can decrease as electric current in the probe gets smaller. In other words, it is expected to make the probe impedance higher, when the length of the probe depth makes small.

The higher limit of probe impedance, however, is restricted by the impedance of the first stage vacuum tube of the null-amplifier.

The relation between the length of depth and the impedance in every material has not been checked. This condition is severe where the automatic control is applied. Therefore, it was studied to get some information about the probe impedance and the polarization of liquid by the A. C. potentiometer. The description of the apparatus is given in Appendix.

II. Electrolytic tank analogue

1. Introduction

It is well known that a physical phenomenon or a engineering problem can be solved analogically by analyzing quite different phenomena, if the same mathematical relation holds. When two physical phenomena A and B can be represented similarly in mathematical from, the phenomenon A can be solved by analyzing B. Typical illustration can be seen in the example shown in Fig. 1. Let us consider the mechanism consisting of the mass which is supported by spring as shown in Fig. 1 a. This system can be solved by analyzing an electric circuit containing inductance, capacity and resistance, as shown in Fig. 1 b.

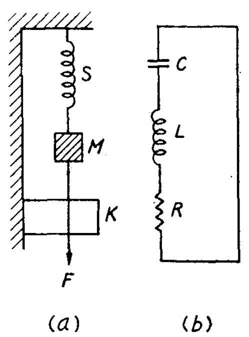

Fig. 1. Mechanical system and analogical electrical system.

Mathematically, these systems can be considered as follows:

Mechanically

$$F = M\frac{d^2x}{dt^2} + K\frac{dx}{dt} + Sx,$$

or

$$F = M\frac{dv}{dt} + Kv + S\int vdt.$$

Electrically

$$E = L\frac{di}{dt} + Ri + \frac{1}{C}\int idt,$$

where mechanical stiffness S, mass M, and friction coefficient K in the mechanical system are equivalent to capacity C, inductance L and resistance R in the electrical system respectively. The mechanical velocity corresponds to the electric current, and the mechanical force to the electrical voltage. In the case of practical measurement, it is difficult to try this experiment by the mechanical system because it is impossible to prepare the spring which has variable stiffness. While it is easy to measure by the electric circuit having variable capacitance. Such an equipment which can be used to solve physical phenomena analogically by quite different phenomena is known as analogue.

The quantities which are represented as the solution of problem by analogue are the physical quantities, i,e, electrical voltage, or current. Thus, the use of analogue is limited in the accuracy of solution. The errors which should be considered in analogue occur (1) when the analogy does not hold and (2) in the measurement. The accuracy is different with the kind of analogue.

In many kinds of the analogue, the electrical analogue is the most useful tool for measurement, because of the easy operation and the simplicity of mechanism.

In the process of development of the electrical analogue, there are two kinds of the use of analogues, i, e, the analogues of the electric field and the electric equivalent circuit. For example, the electric circuit shown in Fig. 1 b. can be considered as the equivalent circuit with the mechanical system in Fig. 1 a. This equivalent circuit analogue, which is discussed by Kron, can be said to be a convenient way to transform a mechanical system into an electric circuit. If the number of elements is increased, however, the circuit becomes much complex. Therefore, the equivalent circuit analogue is considered to be useful for the analysis of the characteristics of a system containing a small number of parameter.

On the other hand, the methods of electric field analogy are classified into the following kinds by the medium in which the electric field formed:

(1) Conducting paper analogue,
(2) Electrolytic tank analogue,
(3) Solid circuit element (containing L and C) analogue.

The first method is used for detecting potential by the electrometer on the conducting paper on which the model electrode is painted by conductor.

This is the extension of the metal sheet analogue which was suggested by G. Kirchhoff. This simple analogue was not in use until about ten years ago. The reason is to be found in the fact that the conducting paper with homogeneous resistance in every direction is not available, and that the probe which is used to detect the potential in the field formed by the paper produces much noise. This analogue, however, has become to be appreciated in the recent year because of the advantage that no polarization occurs near the electrode and the operation is simple(36). The conducting paper to be used has about 1000-5000 Ω/cm^2 which shows considerable homogeneity. This analogue, however, still leaves many problems to be solved in the field where the conductivity shows the directional anisotropy on the conducting paper and a noise occurs near the probe.

The second method is used for forming the potential field by the electrolyte instead of conducting paper. The probe is dipped in the tank to detect the

(36) D.H. Andrews, Electronics, **27** (1954), 182.

potential of model electrode submerged in the electrolyle.(37)

Although this analogue shows the polarization of electrolyte near the electrode, and the mechanical error occurs for the reason that the plotting board and tank are separated, this method is mostly used in the field where the Laplacian equation holds, because of the simplicity of construction and the easy operation,

The third method is that which the electric field is formed by the circuit elements discontinuously rather than continuously by the conducting paper or the electrolyte.

This method has higher accuracy and reliablity than the above two methods, but the mechanism is too much complicated. This method is, therefore, usuful as the computer element of a large scale.

2. General theory of electrolytic tank analogue

The physical phenomena and engineering problems are represented by the following equations:

$$\left.\begin{aligned} \nabla^2 U &= 0, \\ \nabla^2 U &= f(x, y, z), \\ \nabla^2 U &= -\left(\frac{2\pi}{\lambda}\right)^2 U, \\ \nabla^2 U &= \frac{1}{k}\ \frac{\partial u}{\partial t}, \end{aligned}\right\} \tag{1}$$

where, x, y, z, are independent variables, and ∇^2 represents Laplacian operator

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Then let us consider about $\nabla^2 V=0$. So describe generally,

$$\operatorname{div}(PF) = \operatorname{div}(P \operatorname{grad} U) = 0, \tag{2}$$

$$F = \operatorname{grad} U. \tag{3}$$

This is well known Laplacian equation which represents the electric field formed by the electrode, the magnetic field near the poles, the electric current within conductor medium, the mass flow ot hydraurics, the thermal conduction under steady state, and so on.

Let us consider the electric current within the conductor. Then, from Ohm's law,

$$J = \sigma E, \tag{4}$$

where J is current density, E equal to gradient which represnts the electric intensity within the conductor, and σ means the specific conductivity,

$$\operatorname{div} J = \operatorname{div}(\sigma E). \tag{5}$$

Thus, by assuming

$$\sigma = \alpha P, \qquad E = \beta F, \tag{6}$$

(37) (15), (16) cf.

then equations (3) and (5) are coincident, if α and β are assumed to be scaling constants. This field can be used to represent analogically the field where the Laplacian equation can hold.

3. Electrolytic tank analogue

The mechanism which can be used to plot the equipotential line of electric field formed by some type of model electrodes by the electrolytic tank analogue, is shown in Fig. 2. Model electrodes of given type is dipped into the electrolytic

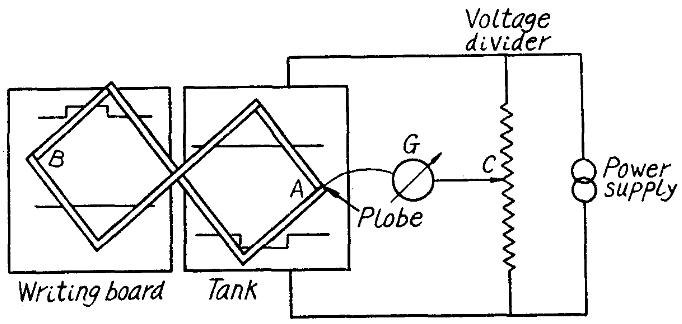

Fig. 2. Schematic diagram of equipotential line plotter.

tank. A bridge is connected by these electrodes, the voltage supply and the voltage divider. The contact of voltage divider C is located at the required reference voltage to plot the equipotential line. One terminal of a galvanometer G is connected to the contact of voltage divider C, and the other terminal leads to the probe, which is fixed on one end of pantograph A. At another end of pantograph B, the pencil which travels on the plotting board is fixed. Thus, since the electrode and the voltage divider form a null detector bridge, the equipotential line of model electrode can be plotted when the probe is moved by the manual operation along the path where a galvanometer current does not flow. In the same way, every equipotential line of a required reference voltage can be plotted by stepping the contact C.

This equipotential line plotter can be considered to be available for the measurement of the field, for example, the analysis of electric field, the design of electronic apparatus, the thermal conduction problem, the stress analysis and so on.

On the other hand, if it is desired to measure the potential at any point in the field,when the probe is submerged at the required point, the contact C represents the potential of the point where the galvanometer current does not flow. One example of this potential measuring problem can be seen in the problem of designing the electrical network function[8],[9][10],[11],[12]. If the electric charge q exists at the origin O in Fig. 3, the potential V at Z can be represented as follows:

$$V = -q \log \rho + \text{const.} \tag{7}$$

Considering by polar co-ordinate,

$$Z = \rho e^{i\varphi}. \tag{8}$$

Therefore, if it is assumed

$$W = V + i\varphi, \tag{9}$$

where V and φ mean the real potential and stream function, respectively,

$$W = -q \log Z + \text{const} \tag{10}$$

will hold. On the other hand, if the charge q exists at Z_m, the above equation will be as follows:

$$W = -q \log (Z-Z_m) + \text{const.} \tag{11}$$

Furthermore, in the case where many charges exist,

$$W = -\sum q_m \log (Z-Z_m) + \text{const.} \tag{12}$$

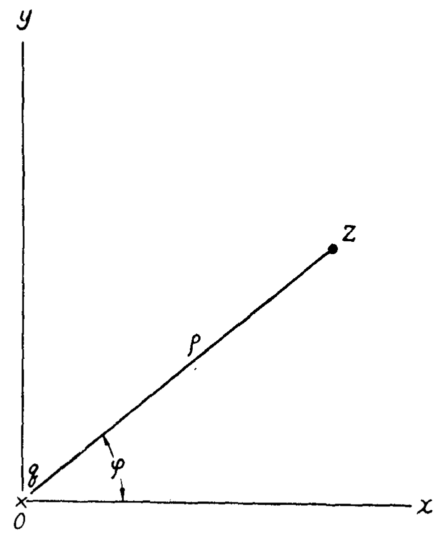

Fig. 3. Explanatory diagram of the relation between electric charge and potential.

Now let us consider the 4-terminal network. The network function can be generally considered as the following polynomial[38].

$$F(P) = \log K \frac{(P-P'_1)\ (P-P'_2)\ (P-P'_3)\cdots\cdots}{(P-P''_1)\ (P-P''_2)\ (P-P''_3)\cdots\cdots} \tag{13}$$
$$= \log K + \sum \log (P-P'_m) - \sum \log (P-P''_m),$$

where P'_1, P'_2, ……, and P''_1, P''_2, …… represent zero and pole of network function respectively. From the analogy of the equation (12) and (13), 4-terminal network function can be represented by electrolytic tank analogue. Fig. 4 shows the measuring method of potential gradient at any point to be required[29]. The second probe is connected to the amplifier A_2. The potentiometer is supplied from a separate winding of a transformer, exactly equal to the other winding. If δV is the potential difference between the probes, a null point is observed, when the

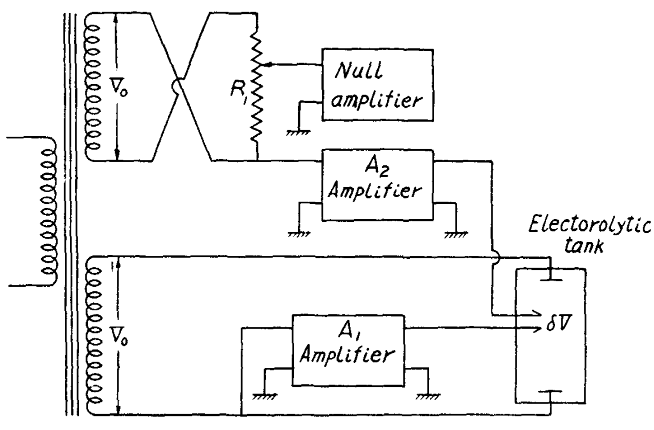

Fig. 4. Measuring Method of potential gradient.

(38) H. W. Bode, *Network Analysis and Feed Back Amplifier Design*, Van Nostrand Co. N. Y. (1945).

potentiometer R_1 reads $g\delta V$ where g is a gain of A_2. Thus the reading of R_1 is a measure of the component of gradient in the direction of the probes.

Combining these method of measuring the potential, the equipotential line and the potential gradient, it is possible to measure another quantity analogically, for instance, an electron trajectory in the electrostatic field.

Thus, the electrolytic tank analogue is a convenient tool for measuring quantities in many fields as described before.

4. Accuracy of the electrolytic tank analogue

The accuracy of the electrolytic tank analogue can be considerad to depend on the following terms:

(1) Perturbation of the field by insertion of probe.
(2) Polarization of electrolyte near the model electrode and probe.
(3) Effect of surface tension of electrolyte.
(4) Mechanical error.

The mechanical error occurs through the presence of the bending of a lever, the backrush of a gear, the friction of the mechanism, and so on. These effects, however, appear remarkablly different in each case, and so it will be described individually in every mechanism.

The perturbation of the field of the model electrodes by insertion of the probe tip occurs in accordance with the high current flow in the probe. The probe impedance must be high enough not to induce the high probe current, by making the length of depth of the probe in the electrolyte small. High probe impedance, however, necessarily causes the high noise level in the output circuit of the probe. This effect is especially remarkable in the case where this probe is connected to the first stage amplifier input of a null detector. Thus, the length of depth of the inserted probe is to be decided by a compromise of the perturbation of the field and the noise level of the amplifier.

Polarization of electrolyte near the electrode remarkablly occurs in the case where the frequency of electrode voltage supply is lower than 1000 cycle.

Since the probe tip in the electrolyte causes the input current of the amplifier (for example, the null amplifier of equipotential line plotter), a perturbation necessarily occurs in the field of the electrolyte.

This effect is remarkable when the two probes (a pair of probe) are used, as in the measrement of potential gradient shown in Fig 4. Furthermore, the depth effect of the probe length is changed according to the kinds of probe metal and electrolyte. The adequate depth of the probe has been reported as 3-5 mm when a nickel coated wire of 0.5 mm in diameter was used, and the electrolyte was tap water. On the other hand, when a platinum wire is used, the length of depth may be fixed shorter than in the case of a nickel wire. Probe impedance depends also on the polarization of liquid and the surface tension. The details of experiment will be described in the other paper.

The polarization of electrolyte near the electrode, makes the analogical circuit, equivalent to that shown in Fig. 5. This equivalent circuit causes the occurrence of error by which the analogical field cannot be formed. According to the previous work, this effect can be reduced to a negligibly small value, by supplying the electrode voltage as high as 1000 cycle in frequency. For practical use, the material which does not induce this effect has to be chosen. For the purpose of the measurement of such polarization, the *A. C.* potentiometer described in Appendix was used. The electrode and electrolyte are prepared in the small cell. According to our experiment, tap water is sufficient electrolyte to make a good field. Although other authers des cribed that tap water is not good medium(8), this difference is due to the composition of tap water in various location.

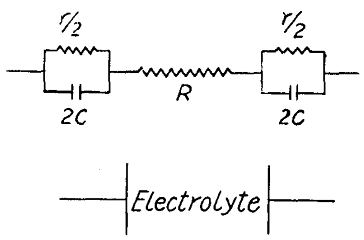

Fig. 5. Equivalent electric circuit of electrolytic tank.

In the process of this experiment, various kinds of metals of electrode were studied, namely, Al, Ni, Ag, Au and graphite coated metal. According to our experimental results, a graphite coated metal is considered to be useful in avoiding this effect. The preparation of the graphite coated metal is that the graphasol is painted homogeously on the base metal and keep at high temperature (300°C) during two hours. As the preparation is, however, difficult for a large type of electrode. the other electrode material is necessary The authors usually use a nickel coated metal for this purpose.

Finally, since the capillary phenomena occur near the electrode and the probe, it induces the perturbation of field. This perturbation is particularly remarkable in the case where the two approached probes are inserted as a tool of the potential gradient measurement and the electron trajectory plotting.

Thus the terms which cause the errors are related to each other. But these errors may be reducedt o some level. The details of experiment to detect these errors will be reported later.

III. Automatic equipotential line plotter

1. Introduction

The methods how to apply servomechanism to the equipotential line plotter by the electrolytic tank analogue may be classified into the following two kinds:

(a) The deviation of the probe from the desired equipotential line is compensated to minimize it as to the normal direction of the line, while the probe is travelling in the tangential direction by constant speed.

(b) By setting X-Y co-ordinate on the plotting plane, the compensation of the deviation of probe from a desired equipotential line is made on the Y-direction, while the probe is travelling in the X-direction at a constant speed(24),(30).

In comparing thes two methods, (a) is considered to be suitable for the application of the servomechanism, when the equipotential line to be plotted is in such a form as closed contour.

Plotting of many successive equipotential lines however, necessitates a manual switching to step the reference voltage divider. The method (b) makes, on the other hand, the automatic switching possible to step the reference voltage divider, although it is necessary to divide the plotting plane into two or more sections for plotting the contour type lines. For this reason, a method of applying the automatic control should be decided by the forms of the model electrodes to be plotted. The authors chose the method (b) in constructing the automatic equipotential line plotter[30].

An example of equipotential line group is shown in Fig. 6. The equipment which can be used to plot these equipotential lines should satisfy the following conditions:

(a) The probe can travel along the all equipotential lines of any type of model electrodes.

(b) Every equipotential line can be traced in a limited domain of electrostatic field (for example, hatched section of Fig. 6)

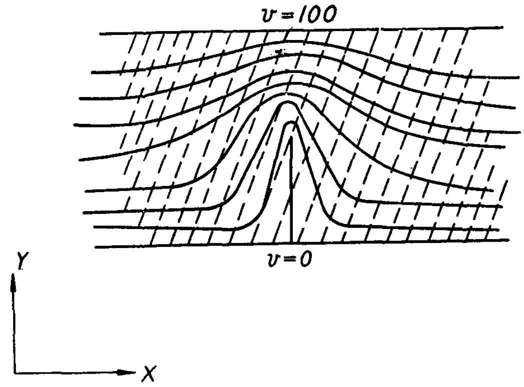

Fig. 6. Explanatory example of electrostatic field.

The equipment which satisfies these conditions are as follows: first, the probe travels at a constant speed to the X-direction in Fig. 6. In this case, the difference between the reference voltage of any equipotential line and the probe voltage induces the current in a galvanometer arm of the bridge circuit shown in Fig. 2.

Driving the servomotor by this current after amplified, the probe always exists on the equipotential line to be plotted, if the polarity of the servomotor is so settled that the deviation current may be decreased. Thus, if the probe approaches the boundary of a limited domain after plotting one equipotential line, the limit switch operates to change the direction of revolution of a X-axis motor, and steps the reference voltage terminal to that of the next equipotential line to be plotted at the same time. Then, the probe travels along the next equipotential line in the X-direction, as in the same way as described above. After approaching the opposite boundary, the limit switch changes again the direction of revolution of the motor and steps the reference voltage terminal. Then the probe plots the corresponding equipotential line. Repeating the same operation, successive equipotential lines can be plotted. After plotting the last equipotential line, this equipment stops at the last boundary.

Photo. 1 shows the overall view of automatic equipotential line plotter, which was constructed on the principle described above.

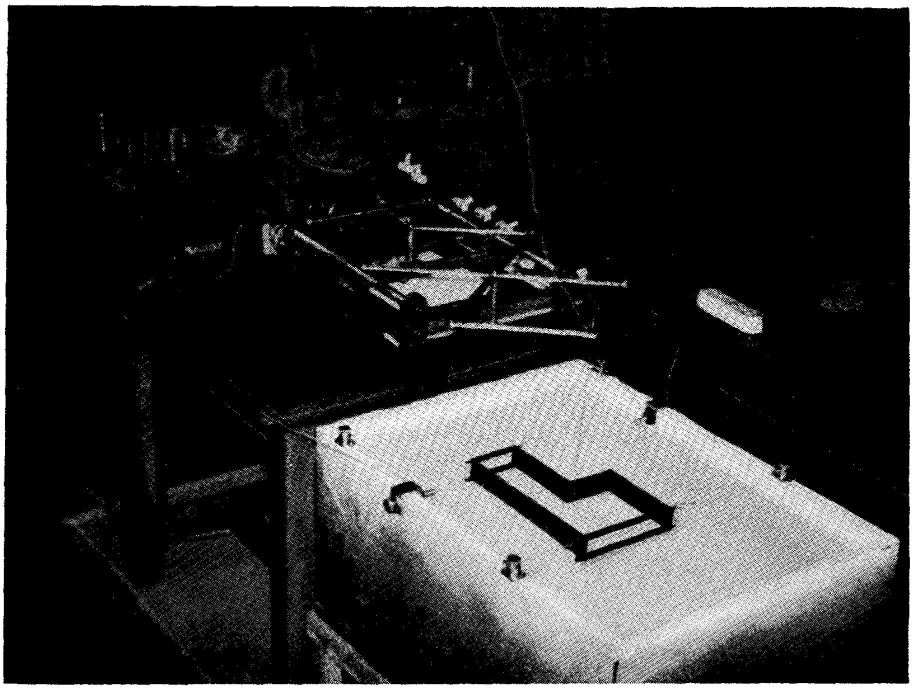

Photo. 1. Overall view of automatic equipotential line plotter.

2. Mechanical construction

The mechanical construction of this equipment is shown in Fig. 7. The coincidence between the position of probe and that of pencil makes possible the control of the probe at the pencil side by the pantograph.

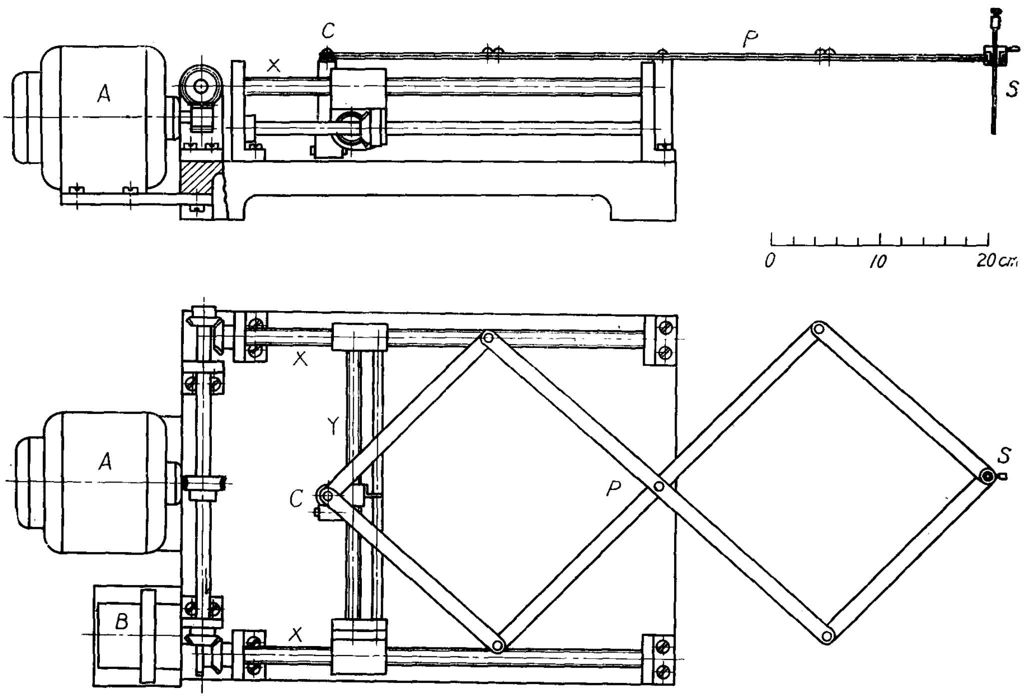

Fig. 7. Schematic diagram of mechanical part, plan and elevation.
A : X-motor B : Y-motor
C : Pen and limit-switch assembly
P : Pantograph S : Probe
X : X-lead screw Y : Y-lead screw

The revolution of the X-axis motor gives the rotations to two herical axis, parallel to the X-axis, which carry one carriage perpendicular to these X-axis and drive it to X-direction at a constant speed. The Y-axis motor gives the revolution to Y-axis which carries the pencil assembly.

The pantograph is made of aluminum as a means to avoid the friction. The material and the form of pantograph should be chosen as to minimize the bending which causes the error of measurement.

Fig. 8 shows the mechanism of the limit switch and pen assembly. A ball pen was used to write successive equipotential lines for a long time, continuouslly, avoiding the leakage of ink. This ball pen is always kept on the plotting board by the spring.

The limit switch operates at the time when the pen approaches to the both boundaries of the domain to be plotted and contacts with the boundary strip. The double springs of this assembly protect the shock of inertia of the revolution of X-motor, before the X-motor is stopped by the operation of relay system. The possible domain of the board to be plotted is 25 × 20 cm².

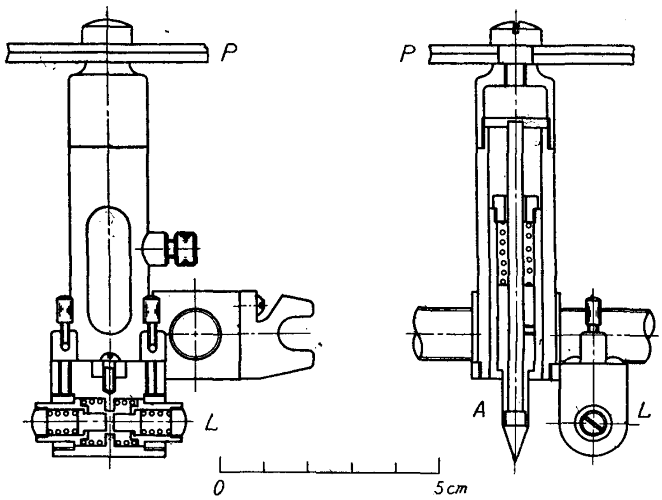

Fig. 8. Assembly of pen and limit-switch.
A : Ball pen
L : Limit-switch
P : Pantograph

A phase splitting induction moter 50 cycle-100 V, 35 watt was used for the X-motor, and the Y-motor is a 100 V 50 cycle-servomotor. The velocity of travelling in X-direction is 5 cm/min. Controlling speed in Y-direction by the Y-motor shuld necesarilly be much more increased, as the travelling velocity in X-direction becomes higher.

The probe is a well-polished cupper wire of 2 mm in diameter which is coated by nickel and is insurated from pantograph by a bakelite washer. The wooden electrolytic tank, $70 \times 50 \times 40$ (depth) cm³, is sheeted by metal. The volume of this electrolytic tank is so large compared with the model electrode that the wall side of the tank does not affect the electric field formed by the model electrode, and the 3 demensional field plotting can be performed[(6)].

The electrolyte in this tank is the tap water which has sufficient conductivity for this purposse.

3. Electric circuit

The electric circuit of this equipment is shown in Fig. 9. The reference voltage of the equipotential line is provided by the voltage divider having nineteen terminals. This voltage divider consists of the series connection of usual solid resistors having total resistance of about 2000 ohms. When the difference between the reference voltage and the probe voltage occurs, this difference voltage is amplified and supplied to the controling coil of the Y-axis motor. Another coil of this motor, the exciting coil, is energized directly by the transformer T. Then, the Y-motor can be so driven as to decrease the deviation current in accordance with the phase shift of these two coils by an adequate choice of polarity.

As illustrated in Fig 9, when the tap of X-motor steps up, the direction of its revolution is changed. The tap (1) (2) ········ forms as tepping relay system which operates as soon as the limit switch is closed. After the completion of the plotting of one equipotential line, then, the limit switch operates to step up the tap of voltage divider of reference voltage. Then, the probe may searches the next equipotential line, and at the same time, the X-motor may reverse the revolution to the opposite direction. Successive equipotential lines can be plotted in this manner.

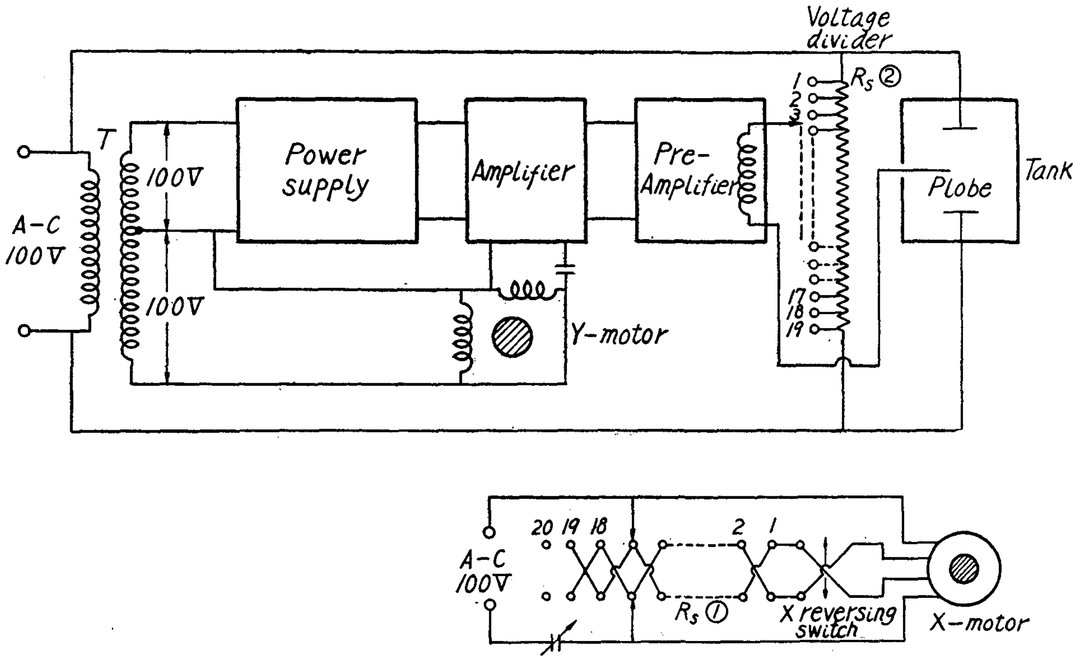

Fig. 9. Diagram of electrical connection.

It is noticed, here, that the stepping up of the tap of X-motor cannot reverse the direction of revolution without bringing the X-motor to complete stop. Furthermore, it is desirable that the X-motor is driven after the probe comes on the next equipotential line, when the voltage divider steps up. For this purpose, a relaying system shown in Fig. 10 was used. The principle of operation is as follows:

1. The limit switch closes when the pen system reaches the boundary.
2. By energizing R_S, the voltage divider and the polarity of the X-axis motor step up. At the same time ,R_T, R_C and R_H are energized to operate the time

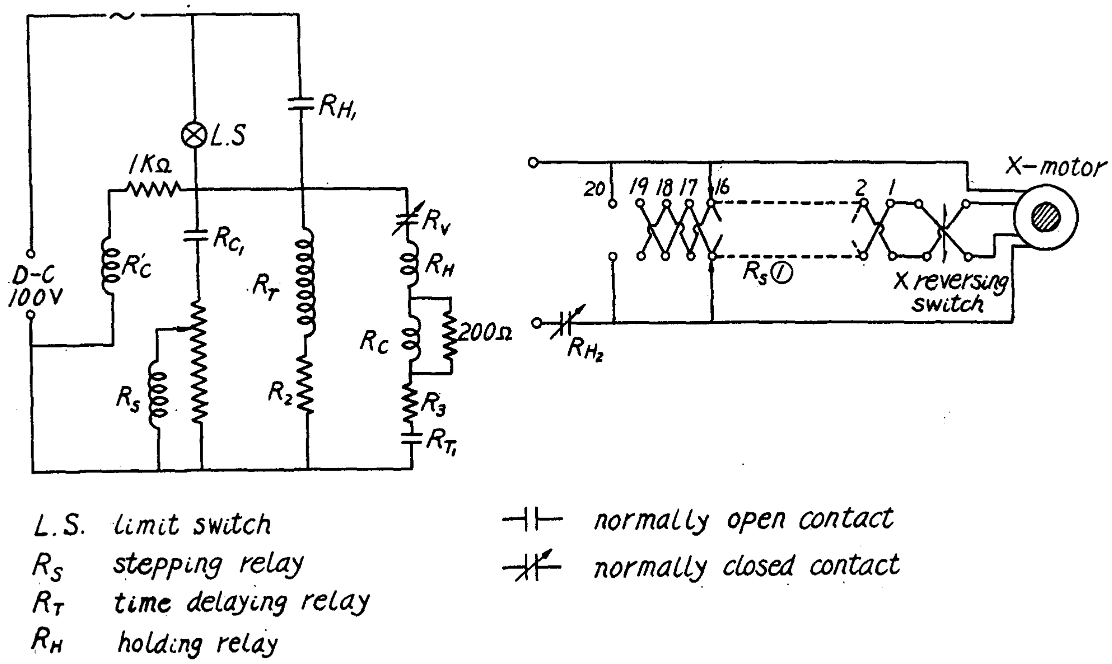

Fig. 10. Diagram of relay system.

delaying relay system.

3. When energizing R_H, the limit switch is sealed by the contact of R_{H_1}, and the X-motor comes to a stop by opening R^{H_2}.
4. By energizing R_C, R_S is de-energized.
5. After a few seconds, R_V is opened and, R_{H_1}. R_{H_2} is closed. Then, the X-motor comes to be driven to the opposite direction.
6. The limit switch opens after pulling away from the boundary. R_{C_1} opens and the condition of circuit is again in the first state.

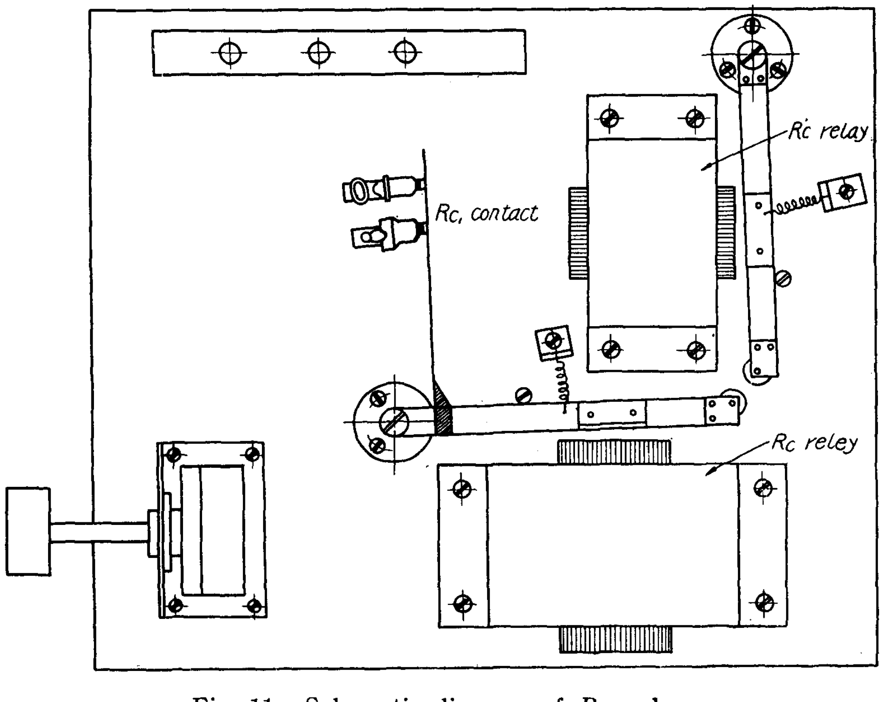

Fig. 11. Schematic diagram of R_C relay.

7. The relay system repeats this operation when the limit switch reaches the boundary until the last equipotential line is plotted.

For the purpose of R_S relay, a telephone rotary line switch was used. Therefore R_S should be energized only for a moment. Furthermore, R_S cannot be energized until the limit switch is pulled away from the boundary, even when the X-motor begins to be driven. To avoid this difficulty, the relay shown in Fig. 11 was used. Here, R'_C is energized directly by the limit switch. In other words, R_S (Fig. 10) is energized only for a short duration between the time of closure of the limit switch and the operation of R_C by energizing R_T (Fig. 10). Furthermore, R_{C_1} remains to be opened, until the limit switch is pulled away from the boundary, even after the X-motor begins to be driven. For the purpose of time delaying relay, an electronic circuit is used. By this circuit, it is available to obtain the time interval between the time closing the circuit and the time passing the plate current through the vacuum tube.

The spark eliminating circuit was inserted in the limit switch circuit to protect the contact point.

Fig. 12 shows the null-amplifier and the power supply. It is necessary that the amplifier is of high sensitivity and of small distortion of the output voltage wave form for the purpose of driving the Y-motor by amplifying the deviation current. The first stage pre-amplifier is set in a separated chasis to avoid noise.

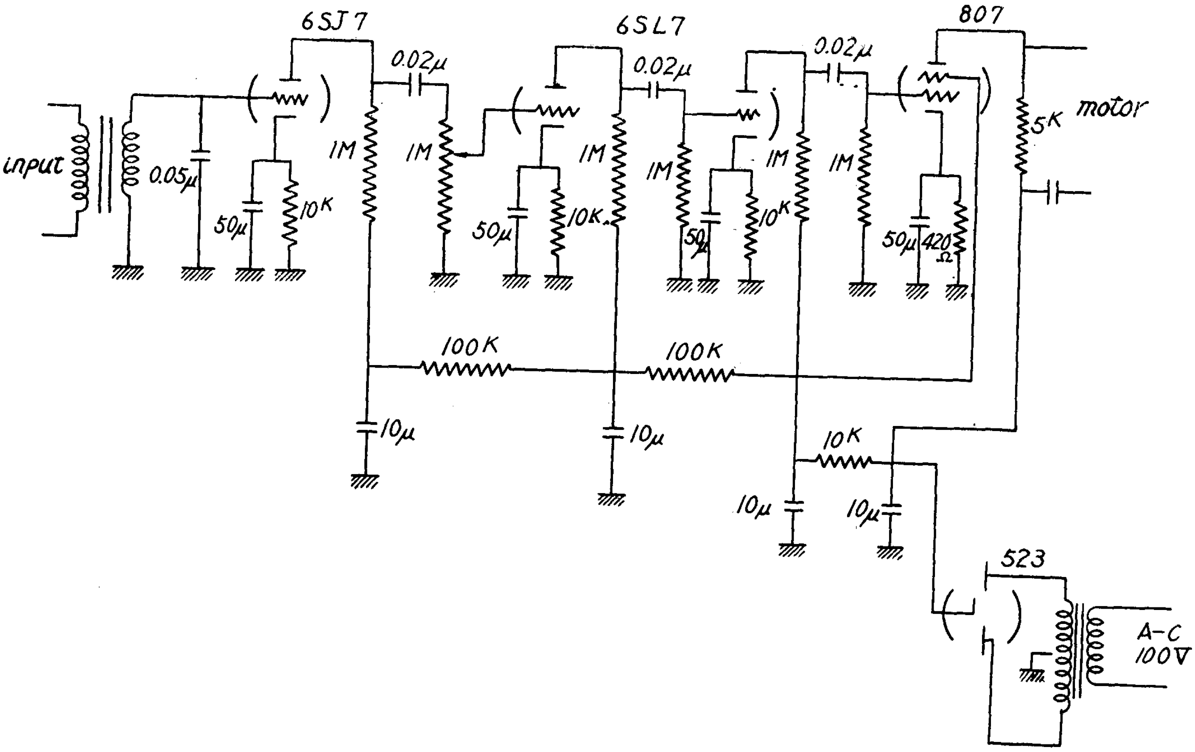

Fig. 12. Schematic diagram of amplifier and power supply.

The voltage supplied to the model electrode is 100 volt 50 cycle. For the purpose of accurate plotting, however, it is necessary to supply a higher frequency voltage than 1000 cycle to avoid the poralization of liquid near the model electrode as described in the last section.

4. Process of operation

The operating process of this equipment is as follows:

(1) The model electrodes are set near the center of the electrolytic tank to avoid the effect of the tank wall on the electric field.

(2) The positional adjustment of plotting board and model electrodes is as follows: First, the corresponding point of pencil side is plotted by bringing the probe to some points near a electrode, and measuring the distance by the divider.

Then, the model electrode is represented on the paper which has been prepared on the plotting board.

(3) The boundary strips on the plotting board are settled at the end of the domain to be plotted.

(4) The voltage divider is set on the first step of reference voltages.

(5) The amplifier, the Y-axis motor, the exciting coil power supply, and the model electrode power supply are switched on, and the probe searches the first equipotential line.

(6) The X-motor is driven after setting the pen on the plotting board.

(7) If the direction of revolution of X-motor is opposite, the X-reversing switch should be turned.

In some cases, it may be necessary that the plotting should be made by dividing the plane into two or more sections when two or more equipotentiallines exist in the same reference voltage or contour type of line. In the other cases, it may be sufficient to plot only half of the field as in the case of the symmetrical form shown in Fig. 6.

5. Experimental results

The examples of plotting equipotential line represented by several types of the model electrodes are shown in Photo. 2, 3, and 4. Photo. 2 and 3 show the perturbation of electric field by the projection in a parallel wave guide. Photo. 4 shows the stream line of perfect fluid in a suddenly constricted channel. The ripple of plotted curve is the important factor to decide the accuracy of measrement. The formation of ripple may be considered as follows:

(1) Friction of a mechanical construction produces a dead band which cannot be controlled by small deviation currents. This dead band remains as a residual error. Because of an increase in the stability of the mechanism and the convergency of hunting, friction may be allowable so long as accuracy is kept. By plotting the equipotential lines of the same reference voltage to X and $-$X-directions, it was found that the difference between these two plotted lines was as small as the thichness of the pen trace.

(2) The plotting of the section near the electrode can be done continuously at comparatively low sensitivity, while the plotting line comes to a stepped form as recede from the electrode. This is due to the fact that the gradient of electric

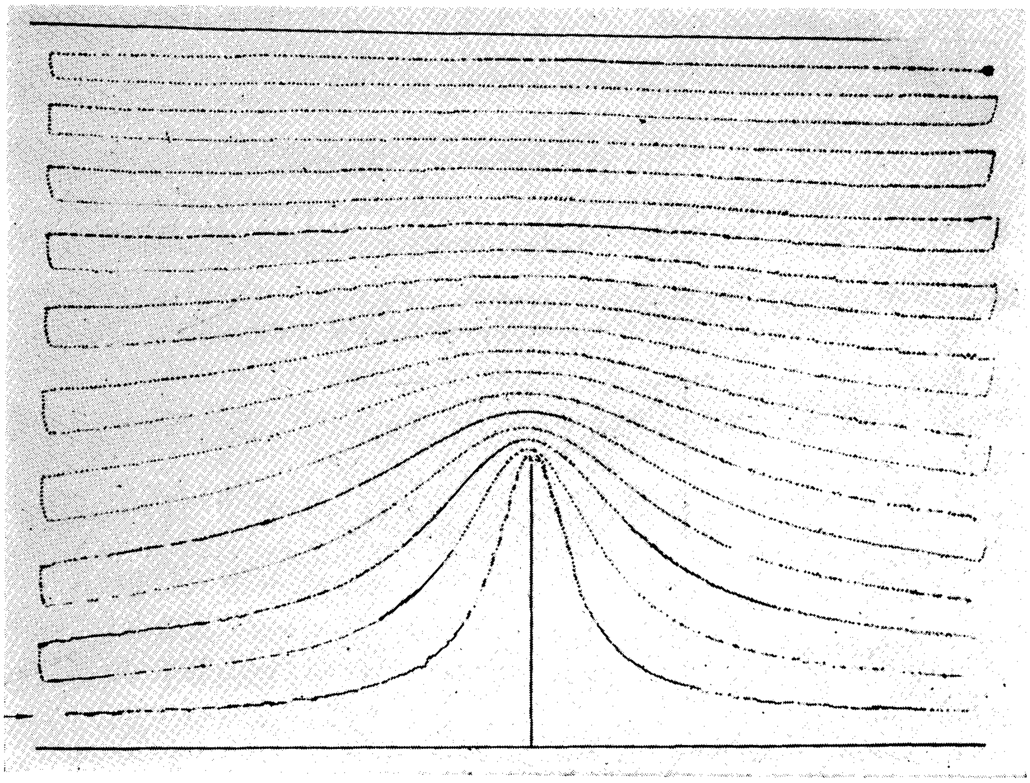

Photo. 2. Perturbation of the electric field by the projection. (× ½)

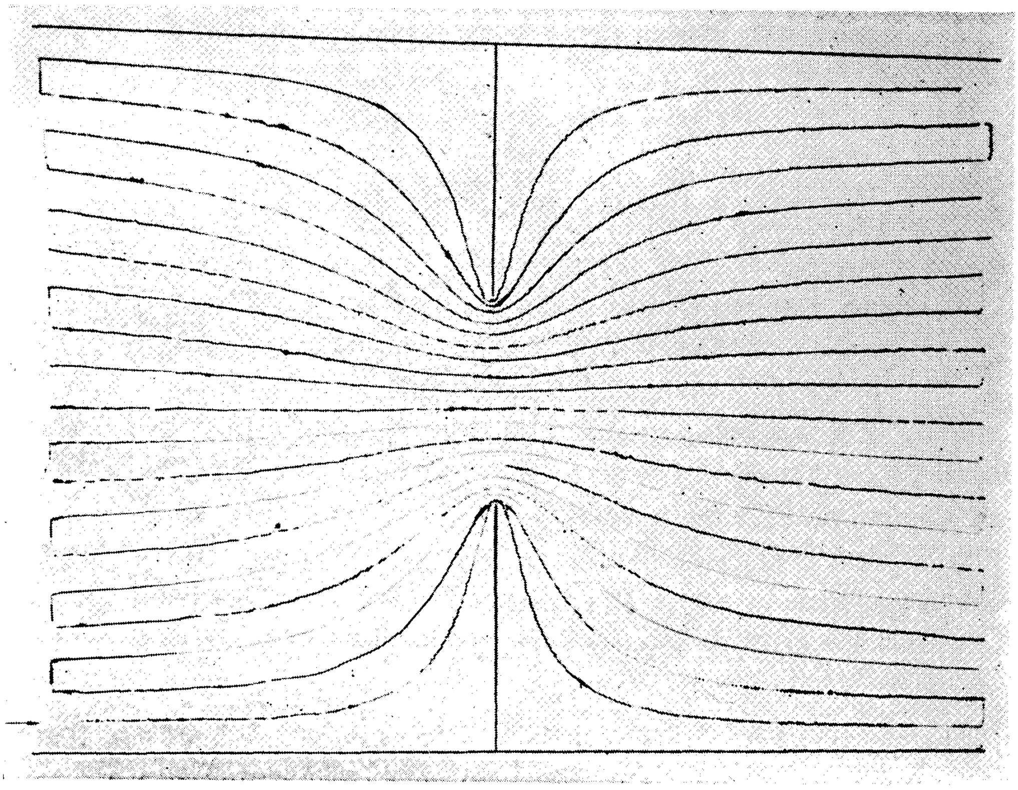

Photo. 3. Perturbation of the electric field by the projection. (× ½)

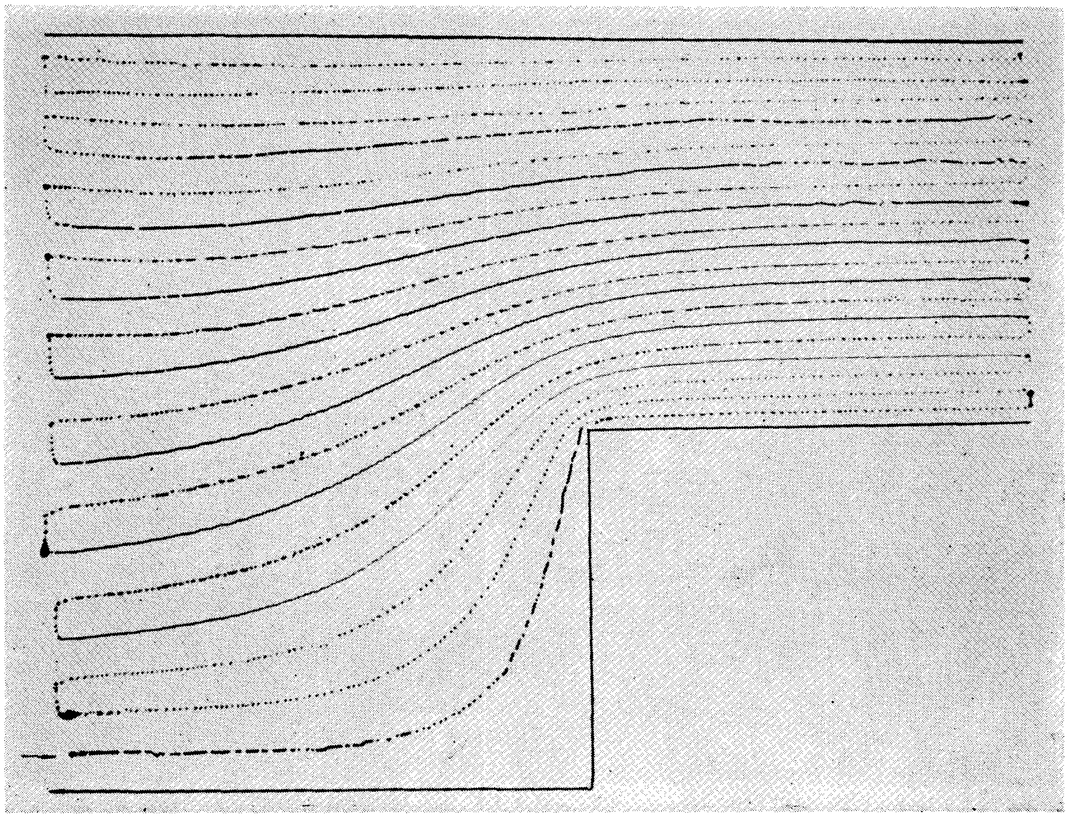

Photo. 4. Stream line of the perfect fluid in a suddenly constricted channel. (×½)

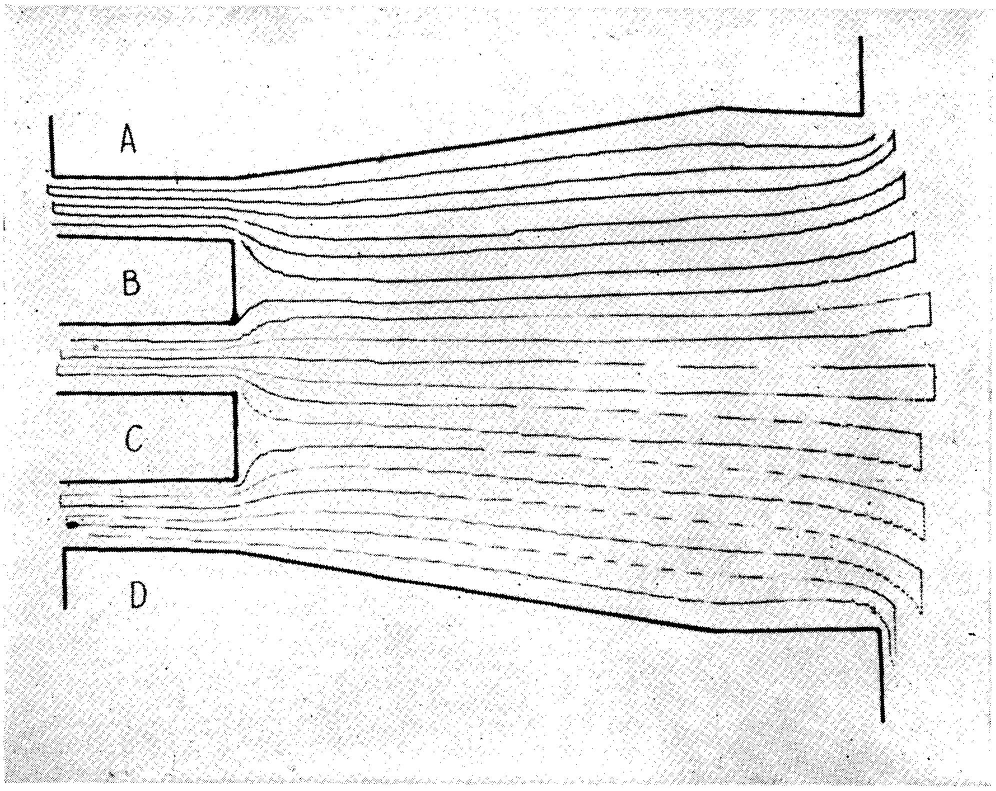

Photo. 5. Distribution of magnetic field extremely near the magnetic poles used in betatron. (×½)

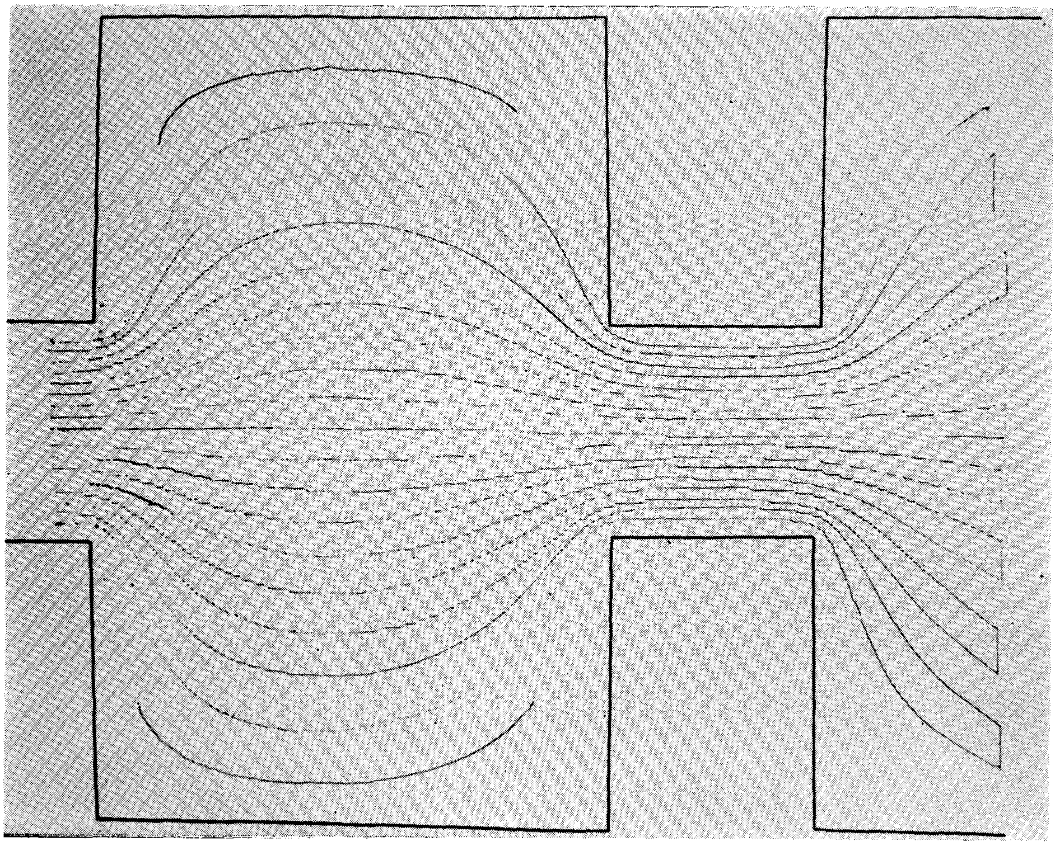

Photo. 6. Distribution of magnetic field extremely near the magnetic poles used in betatron. $(\times \frac{1}{2})$

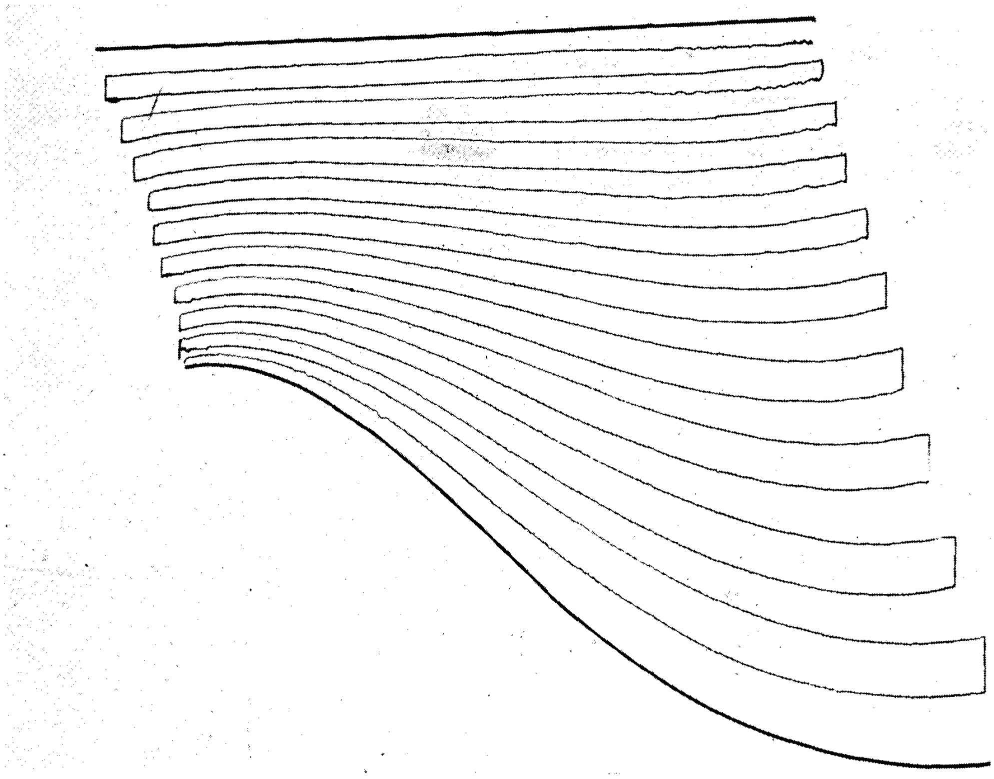

Photo. 7. Distribution of electric field of the electron gun used in linear accelerator. $(\times \frac{1}{2})$

current is different in the sections described above. For this reason, the depth a of submerged probe should be fixed at 4–5 mm or more, otherwise the perturbation of the field may be emerged.

(3) Phase splitting condenser of the Y-axis motor has to be determined at an adequate value, otherwise hunting may be occurred.

(4) With reference to the accuracy of the mechanism, the bending of pantograph and the backrush of bevel gears are important factors for overall accuracy.

It is necessary that the accuracy should be higher by the adjustment of the construction and the improvement of electrode supply to 1000 cycle or more.

6. Application to the design of an electronic equipment

Photo. 5 and 6 show the data for the design of a betatron obtained by the equipotential line plotter. The model electrodes shown in Photo. 5 represents the magnetic field extremely near the magnetic pole in which the electrolytic tank analogy can be applied. The voltage supplied to the electrodes A. B. C. D. are 0. 100/3, 100/1.5 and 100 volts. A 2 KW, 200 V Ni-Cr alloy heater line was used for this voltage divider.

Photo. 7 shows the electric field distribution of an electron gun reported by Pearce which is used in a linear accelerator.

IV. Automatic electron trajectory plotter

1. Introduction

The design of electronic vacuum tube or electron lens of electron microscope requires the knowledge of electron trajectory in the electrostatic field. Since mathematical calculation necessitates the laborious work and is almost impossible to solve problems as the electrode system becomes much more complex, therefore, experimental methodes are necessary.

The method of measuring electron trajectory in two demensional electrostatic fields by an electrolytic tank, which was introduced by D. Gabor[14], is based on the fact that when a pair of wire probes dipped into the electrolytic tank detects ε_n (The potential gradient normal to the direction of motion) and V (the potential), the radius of curvature of electron trajectory is equal to the ratio $2V/\varepsilon_n$. The plotting is achieved by the continuous adjustment of the radious of curvature of the probe trajectory compensating the deviation, as the probe travels on the suface of electrolyte. D. Langmuir[20] suggested the possibility of the automation of this method, and furthermore, several authors reported on the instruments which realized the automatic measurement. After the succesecive improvements by several authors, the equipment described here can be used for plotting the electron trajectory with the model of electrode.

2. General methods of plotting electron trajectory

The method of plotting the electron trajectory in electrostatic field can be classified into following two kinds:

(a) The motion of electron in the two-demensional electro-static field can be represented by the following equations:

$$m\frac{d^2x}{dt^2} = -e\,\varepsilon_x\,,\qquad m\frac{d^2y}{dt^2} = -e\,\varepsilon_y\,, \tag{14}$$

where m is the mass of electron, t the time, x, y the displacements of the corresponding direction, e the electron charge, and ε_x, ε_y represent the potential gradient in x- and y-direction respectively. Equation (14) can be rewritten in the integral form as follows:

$$x = \int_0^t \left(u_0 - \frac{e}{m}\int_0^{t_1} \varepsilon_x\, dt_2\right) dt_1,\qquad y = \int_0^t \left(v_0 - \frac{e}{m}\int_0^{t_1} \varepsilon_y\, dt_2\right) dt_1, \tag{15}$$

where u_0 and v_0 represent the initial velocity component of the electron. The potential gradients of x, y-direction in any point, ε_x ε_y, can be detected by the two pairs of probes dipped in the electrolytic tank, when these pairs are put in the position normal to one another. By converting these values to the rotation of mechanical shaft, that is, by integrating these values twice with time, it is possible to represent the position of the electron trajectory at any moment. Therefore, the electron trajectory can be traced as the continuous line of the points corresponding to x, y described above(31).

(b) Let the velocity of electron and the electric potential at a position where the electron exists, be v and V respectively, equation (14) can be rewritten as follows:

$$Ve = \frac{1}{2}mv^2. \tag{16}$$

This relation can be represented by poler co-ordinate as follows:

$$\varepsilon_n e = mv^2/r, \tag{17}$$

where ε_n represents the potential gradient of the position where the electron exists and r is the radius of curvature of the electron trajectory at that point.

From (16) and (17), we have

$$r = 2V/\varepsilon_n.$$

This means that the radius of curvature of the electron trajectory at any position is twice the ratio of the electric potential gradient at any point.

Thus, the electron trajectory can be traced as the continuous curve having the given radius of curvature at successive moments, if these two values V and ε_n are detected by the probes and converted to the real radius of curvature by an adequate mechanism by supplying the voltage to the model electrodes in an electrolytic tank(21),(22).

In comparing these two methods described above, (a), for example, makes possible the plotting of trajectory of a small radious of curvature, although many probes are needed and the mechanical integrating system must be prepared.

The method of plotting will, then, be decided by the type of electrodes. The present authors have selected the method (b) for constructing the electron trajectory tracer, improving the previous work.

3. Principle of automatic plotting

As shown in equation (18) in the last paragraph, any electron trajectory in the electrostatic field can be traced by the conversion of $2V/\varepsilon_n$ to the form of radius of curvature of the trajectory after detecting $2V/\varepsilon_n$ in every position where the electron exists. The principle of the method for detection and control of the ratio $2V/\varepsilon_n$ is shown in Fig. 13 as a bridge circuit of V and ε_n. If it is desired that no current flows in a null-amplifier arm, the following condition must be satisfied:

$$\frac{\alpha V}{b\beta\varepsilon_n}=\frac{g-y}{y}, \tag{19}$$

where α; gain of V voltage amplifier
β; gain of ε_n voltage amplifier
b; distance of two probes to detect ε_n

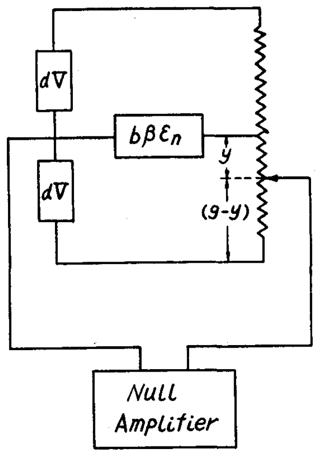

Fig. 13. Explanatory diagram of detecting $2V/\varepsilon_n$.

The relation (19) can always be satisfied, whenever the contact point of potentiometer shown in Fig. 13 is controlled by the servomotor which is driven by the amplified current of null amplifier arm, selecting the polarity adequately. As the apparatus which is used to convert the value $(g-y)/y$ to the radius of curvature r, a tricycle trolley shown in Fig. 14 may be available.

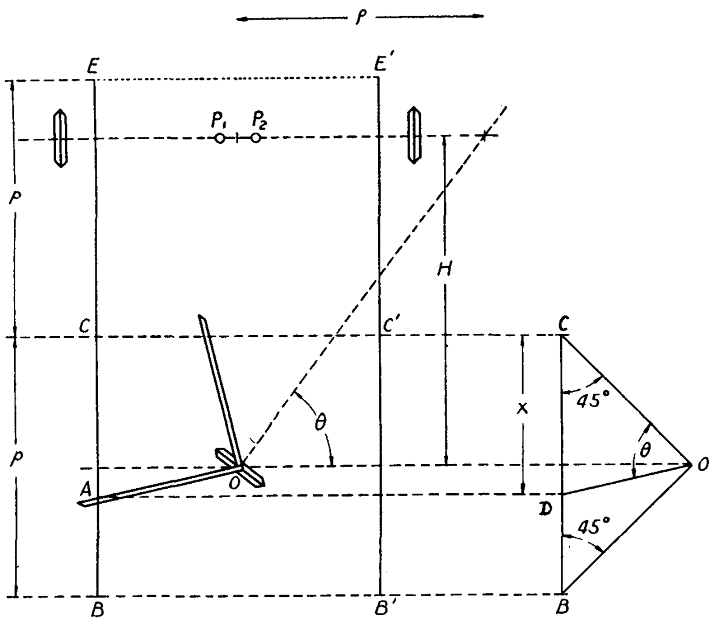

Fig. 14. Schematic diagram of trolley.

Putting the angle of the front wheel of this trolley as θ, the radius of curvature ρ can be represented by the following relation:

$$\tan\theta = H/\rho. \tag{20}$$

If we consider the right-angled equilateral triangle shown in the right hand of Fig. 14, the following geometrical relation can be saticified as $\angle$COD $= \theta$:

$$\tan\theta = \frac{x}{p-x}. \tag{21}$$

Comparing this relationship with equation (19),

$$\frac{y}{(g-y)} = \frac{x}{(p-x)} \tag{22}$$

will hold. Thus, if the relation

$$\frac{2}{\rho} = \frac{2}{H}\frac{x}{(p-x)} \tag{23}$$

holds, and the two probes are positioned just on p_1 and p_2 respectively, the electron trajectory can be plotted at the center of $p_1\,p_2$, by pulling the trolley. In this case, the positions of the pencil and the probe are coincident, and BC and B′C′ are represented as the half part of the potentiometer. The overall performance of the equipment is shown in Fig. 15. The procedure of the equipment can be explaned as follows: First, the voltage is supplied to the model electrode submerged in the electrolytic tank. A pair of probes detect the two voltages V and ε_n.

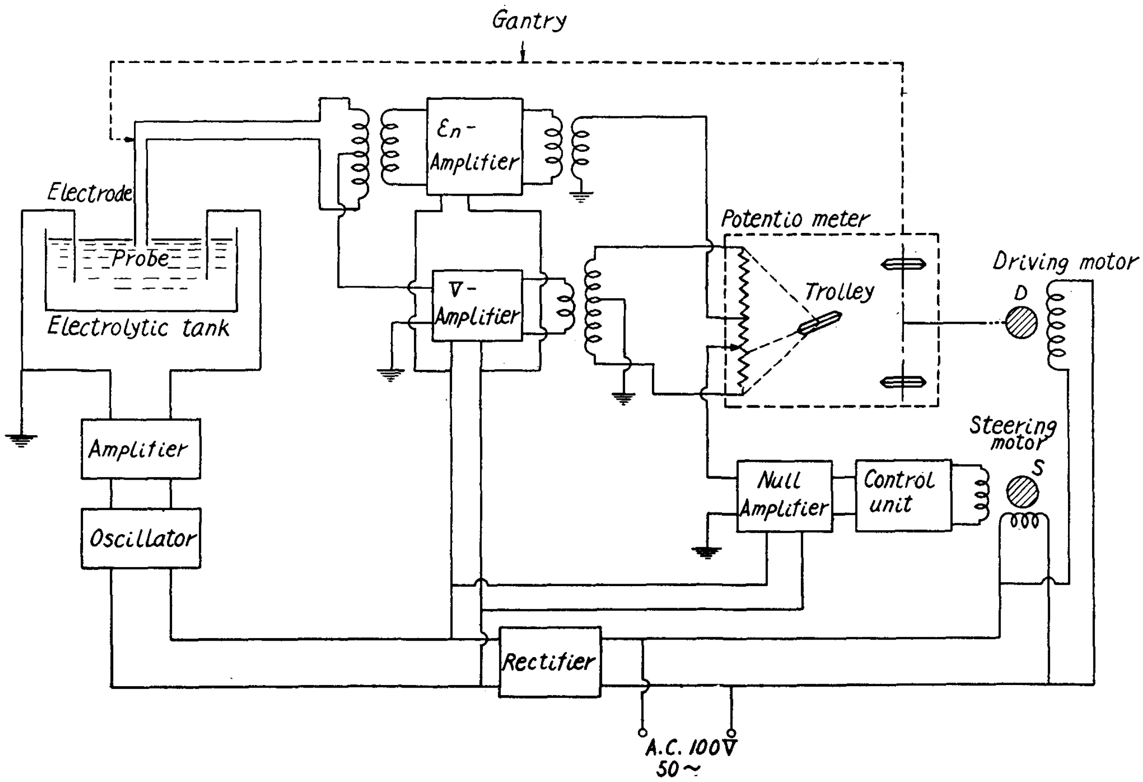

Fig. 15. Schematic diagram of overall performance of the equipment.

These voltage terminals of V and ε_n were combined to the potentiometer to form the bridge circuit. The trolley is pulled by the driving motor D, and the steering system of the trolley is connected mechanically with the potentiomenter. The pair of probes are positioned in the center of the shaft of the back wheels of the trolley and are to be coincident with the position of the pencil by the gantry G. which travels along the long side of tank. Photo. 8 shows the overall view of the equipment constructed on the principle described above.

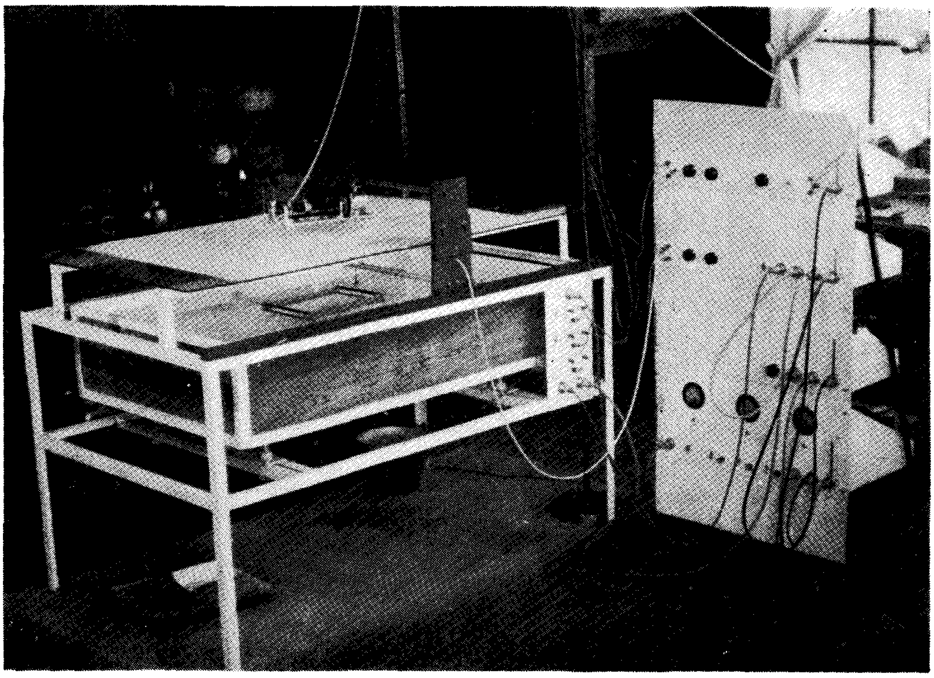

Photo. 8. General arrangement of tank and control circuit.

4. Automatic plotter

(i) Electrolytic tank and probe

The wooden electrolytic tank covered by the sheet metal is as large as $120 \times 70 \times 30$ (depth) cm. Model electrodes are hung from the wall side of the tank. The electrolytic tank is made so large, compared with the model electrode that the wall side may no longer affects the electrostatic field. Moreover, three-dimensional field problem may be analyzed(6) by this apparatus.

The electrolytic tank is supported by four jacks to adjust the level of electrolyte and is possible to incline at will for the measurement of three-dimensional fields.

Since the pair of probes detect the voltages V and ε_n on the surface of the electrolyte, they should always travel just on the surface of electrolyte to keep the constant dipped length in the electrolyte. For this purpose, the gantry is constructed which travels on the rails parallel to the long side of the frame of electrolytic tank. Two pairs of rails are fixed on the gantry. The upper pair is to carry the trolley and the lower pair is to carry the pair of probes respectively. This gantry is driven by the motor D in Fig. 15. The unit of setting probes are

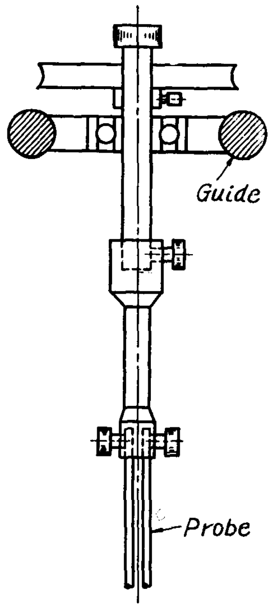

Fig. 16. Unit of setting probes.

shown in Fig. 16.

The probe, namely two well polished nickel-coated wires of 0.3 mm in diamenter fixed at the distance of 4 mm, is kept in the position normal to the surface of the electrolyte.

On the upper location of the tank, a well-polished glass sheet of 6 mm in thickness is fixed, on which the tracing trolley runs. The parallelism of this glass plate with the surface of electrolyte was determined by a level meter.

As the electrolyte, tap water with sufficient conductivity was used as described in the section of equipotential line plotter.

(ii) Power supply and oscillator

If the low frequency voltage is supplied to the model electrodes which were dipped in an electrolytic tank, a homogeneous electrostatic field cannot be formed

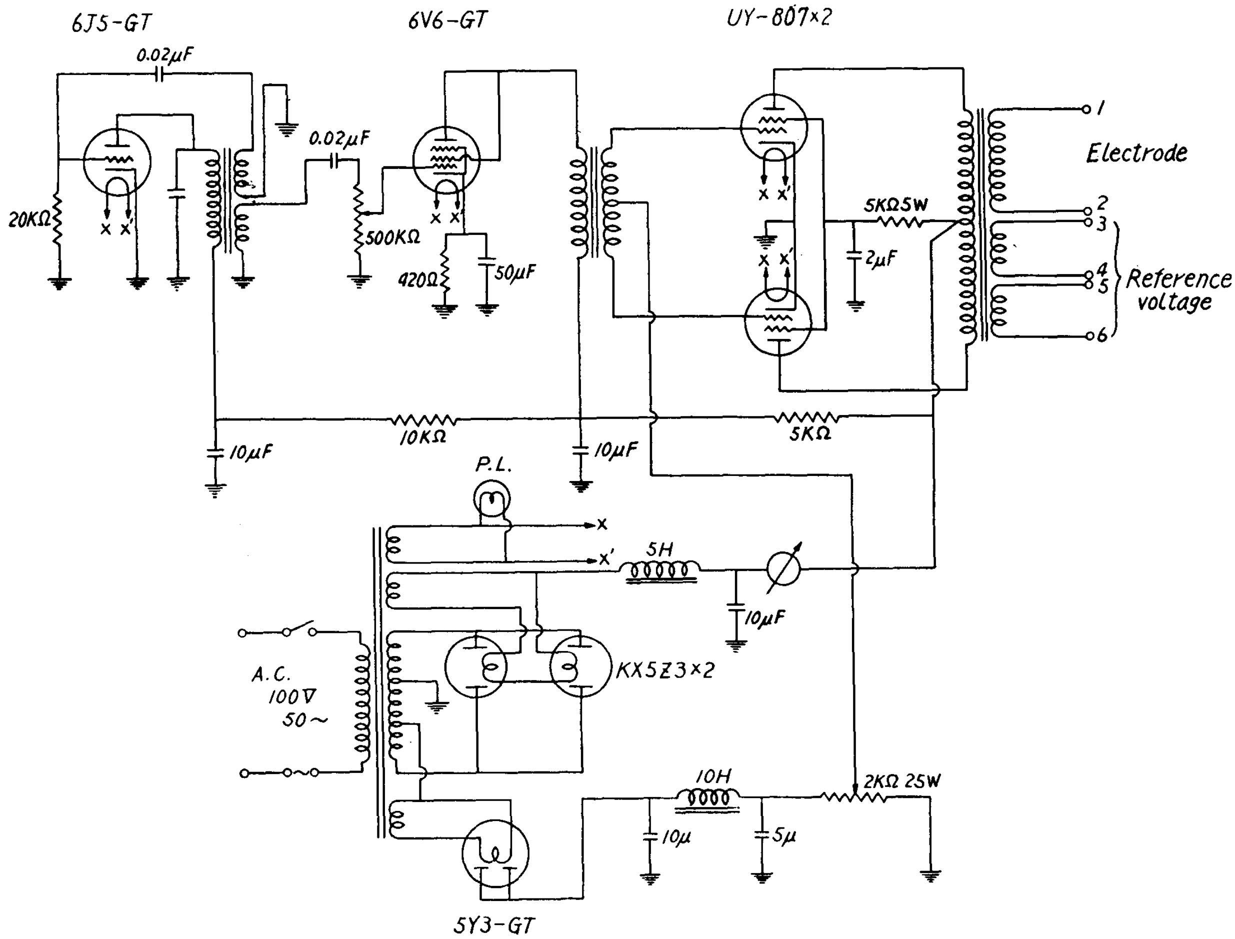

Fig. 17. Schematic diagram of oscillator circuit.

analogically because the polarization of liquid occurs near the electrodes.

According to the relation between the polarization and the frequency of supplied voltage of electrode, about which G. Jones[34] and G. Jaffe[35] reported, the polarization will decrease suddenly at voltage higher than 1000 cycle in frequency. For this reason, a vacuum tube oscillator .of the 1500 cycle frequency was used. The schematic diagram of the oscillator circuit is shown in Fig. 17.

Since this oscillator also plays a role of providing the reference voltage for detecting the phase shift of the motor control unit as well as supplying the voltage to the model electrode, the wave form of voltage must be considerablly sinusoidal. The output power of the oscillator is about 40 w because the amplifier of A B_2 Class is used. The level of supplying voltage of model electrodes can be changed by switching the output terminal of the tansformer. The power supplies of the oscillator, and the V, ε_n amplifiers were constructed in the same pannel which has the separated chasis, together with the steering motor control unit.

(iii) Potential and potential gradient amplifiers

The amplifiers of the potential and the potential gradient are shown in Fig. 18. Since the probe impedance is to be kept at high level to avoid the perturbation of electric field, the input current of these amplifiers is so small that the current

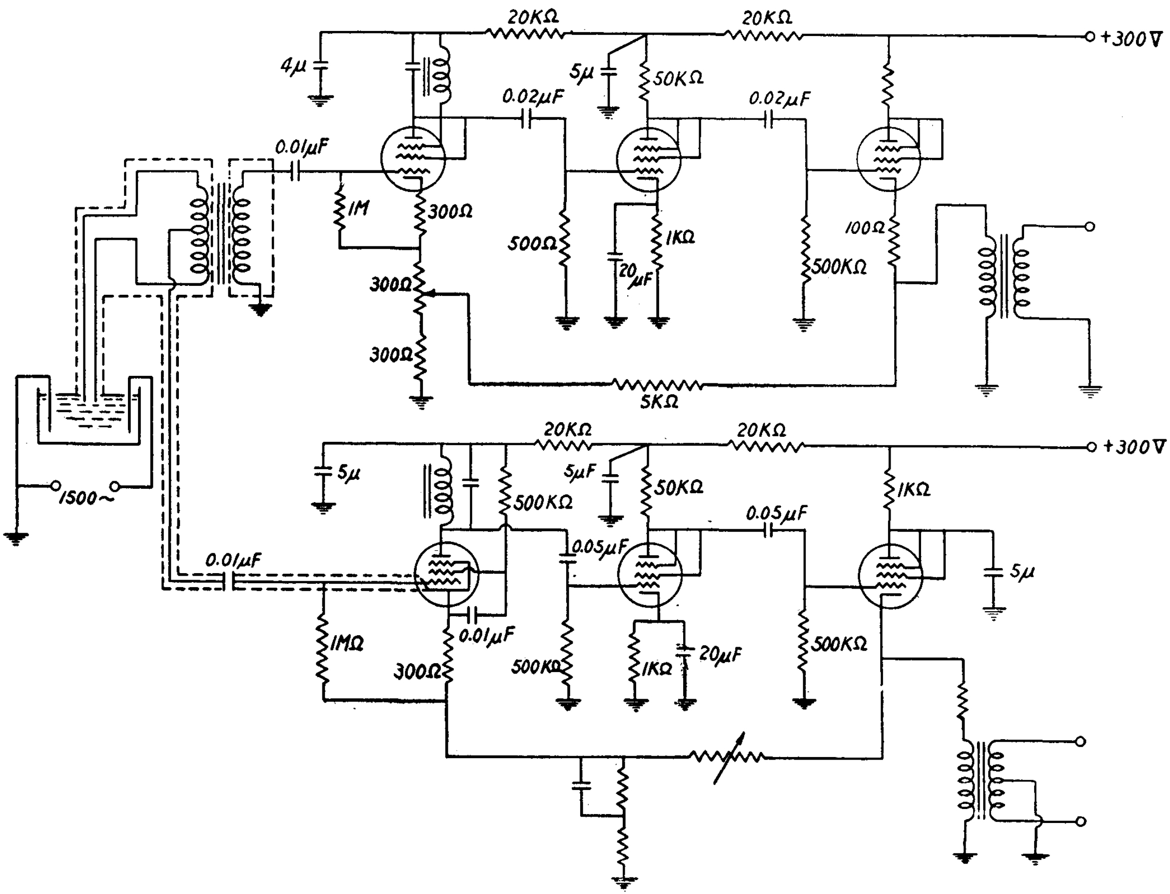

Fig. 18. Schematic diagram of V and ε_n amplifier.

amplification may be needed. Here, the difficulty occurs as follows: In Fig. 19 (a), M represents the position of the centre where the pair of probes exist. When the point M is not in coincident with the grounded point S, the equivalent bridge curcuit shown in Fig. 19 (b) is formed, where R_1 and R_2 represent the resistances of liquid between both probes and M. C_1 and C_2 are the distributed stray capacity of the transformer to the earth respectively. Then the voltage difference between M and S produces the voltage at the output terminal of the transformer. This effect makes impossible the accurate detection of potential gradient by the probes. To compensate this error, the factors, C_1, C_2, R_1 and R_2, must be fixed to balance the bridge circuit. Since the position of pair of probes is continuouslly changed, it cannot be helped to satisfy these conditions.

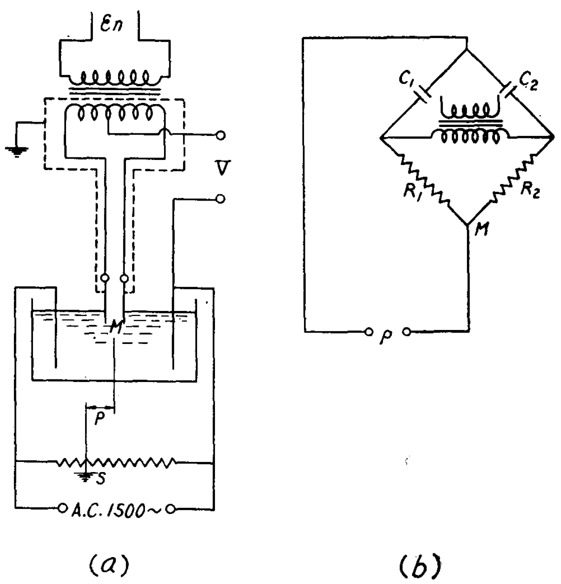

Fig. 19. Equivalent circuit diagram of input circuit containing stray capacity of input transformer.

For the purpose of avoiding this difficulty, it is necessary to change the position of the earth point by driving the motor, as the pair of probes changes the position. This mechanism, however, makes the system too complex. Instead of using the motor, an accurate measurement can be achieved by connecting the shielding of the primary winding of the input transformer to the point where the potential is very close to that of the pair of probes rather than to the earth. Thus, the shield was connected to the cathode of the potential amplifier, as shown in Fig. 18. Since the V and ε_n amplifiers supply αV and $b\beta\varepsilon_n$ to the bridge circuit shown in Fig. 13 respectively, they must be of constant gain on input voltage and independent of time. Therefore, the both amplifiers are kept stable by applying the cathode follower system and the negative feedback. In the potential amplifier, the screen grid of the first stage tube is connected to the cathode rather than to the earth to avoid the blocking oscillation.

The power supplies of both amplifiers are kept at constant voltage by the voltage regulator to keep the constant gains of the amplifiers.

(iv) Null amplifier and motor control unit

In the bridge circuit shown in Fig. 13, the current, which flows in the null amplifier, is supplied to drive the steering motor after amplified, to minimize itself by the adequate choice of the polarity by changing the contact point of potentiometer. This null-amplifier of four stages is shown in Fig. 20. The null-amplifier output of 1500 cycle in frequency cannot be supplied directly to the control coil of the steering motor as it is driven by the voltage of 50 cycle in frequency.

Since the output current of the amplifier is supplied to the steering moter control unit, it is necessary to keep the output voltage at constant level. For this purpose, an automatic volume control system was applied to suppress the amplifier gain.

The deviation current after amplified by the null-amplifier is supplied to the motor control unit shown in Fig. 21. Depending on the direction of deviation of the contact point of potentiometer from the balancing point, a phase difference of null-amplifier current becomes 180°. For the purpose of detecting the direction of deviation, therefore, an output of the null-amplifier is compared with the reference voltage of the main oscillator. If in phase, both are added and a grid voltage of

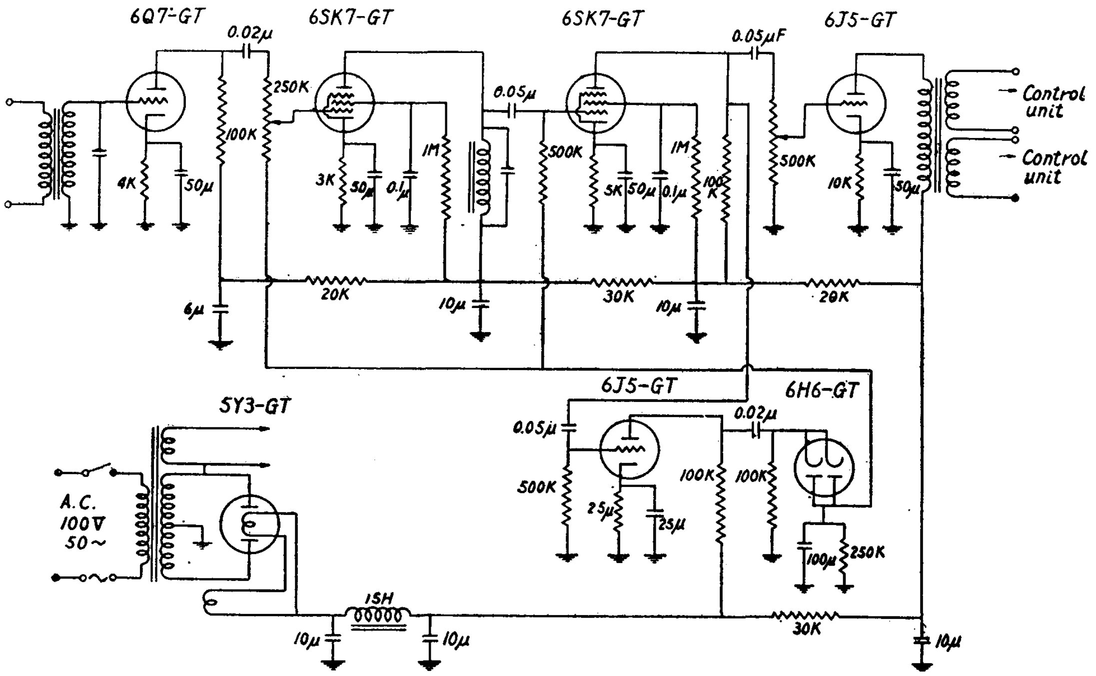

Fig. 20. Schematic diagram of null amplifier.

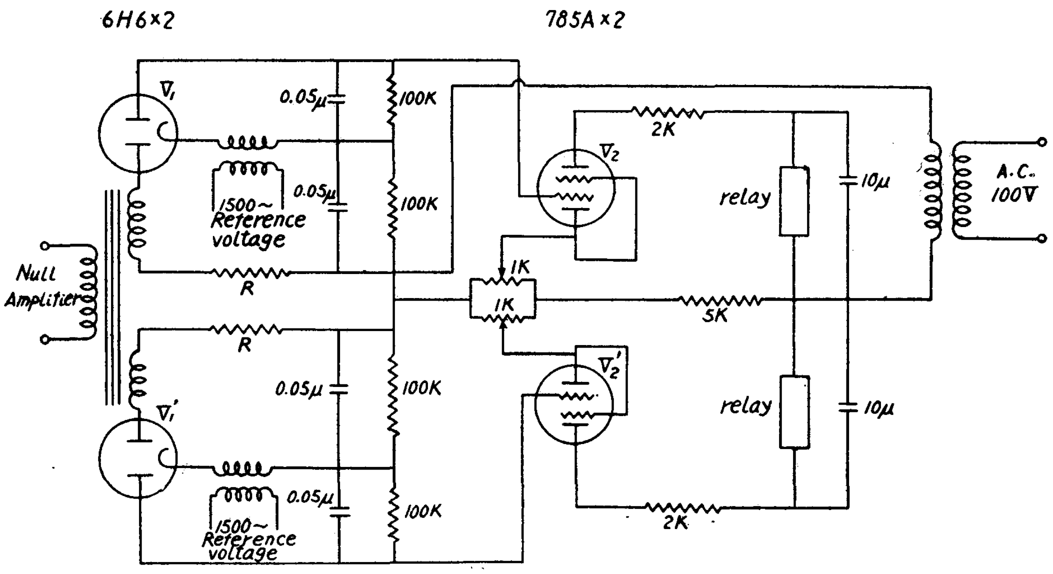

Fig. 21. Schematic diagram of control circuit.

one of thyratron tubes (785A) becomes higher after the rectification by the tube *6H6-GT*. In this case, the reference voltage provided from the main oscillator does not affects the grid voltage by combining the bridge circuit with the output of the null amplifier as shown in the diagram. If the phase is reversed, on the other hand, the same phenomenon occurs in the other bridge circuit. In other words, one thyratron tube fires depending upon the in phase deviation current. Thus, the firing of thyratron tube energizes the relay inserted in the anode circuit, and drives the motor, selecting the polarity to decrease the null-amplifier current. The relays used operate at 24 V and 10 mA. The voltage of null-amplifier output and the reference voltage have to be just in phase or 180° out of phase, depending upon the direction of deviation of the bridge balancing. The phase difference from in phase or 180° out of phase causes the error in balancing the bridge circuit. This was examined by the *A. C.* potentiometer which will be described in Appendix.

It should be noticed that simultaneous firing of both thyratron tubes must be strictly avoided. As shown in Fig. 21, the resistance R prepares the dead band in the bridge circuit and protects this danger by keeping away the firing voltage of both thyratron tubes.

(v) Tracing trolley

For the purpose of plotting the electron trajectory as a continuous curve, a tricycle type of trolley was used which converts the detected value of $2V/\varepsilon_n$ to the radius of curvature shown in photo. 9. This trolley is driven by the driving motor of the single phase induction type. (See the right and back side of trolley shown in Photo. 9).

The velocity of trolley is 50 mm per minutes. The driving motor of the trolley also drives a gantry which will be described later. The potentiometer windings of the bridge circuit shown in Fig. 13 are fixed at the both sides of the trolley (Photo. 9). These windings are Ni-Cr alloy wires of 0.1 mm in diameter, which are wound helically around half of two ebonite cylinders of 10 mm in diameter. Only the upper half section of the potentiometer (shown in Fig. 13) is wound around a half of one ebonite cylinder and the remaining lower half section is formed in the same way on the other cylinder. These two parts of the potentiometer are fixed in parallel at the both sides of the trolley as illustrated in Photo. 9.

If the current flows in the null-amplifier arm, the rotation of steering motor (photo. 9) gives revolution to the two screws which are parallel to the potentiometer and carry the contact point at the potentiometer. The steering motor is a 1 watt servomotor. In this case, if the contact point (in Fig 13) moves lower, the contact of the other potentiometer moves away to the opposite direction by detaching from the winding. This contact mechanism pulls the L type lever which is mechanically connected to the rotating shaft of the front wheel.

Therefore, the rotation of the front wheel is given by the servomotor in connection with the contact mechanism of potentiometer. If the contact A (Fig. 14)

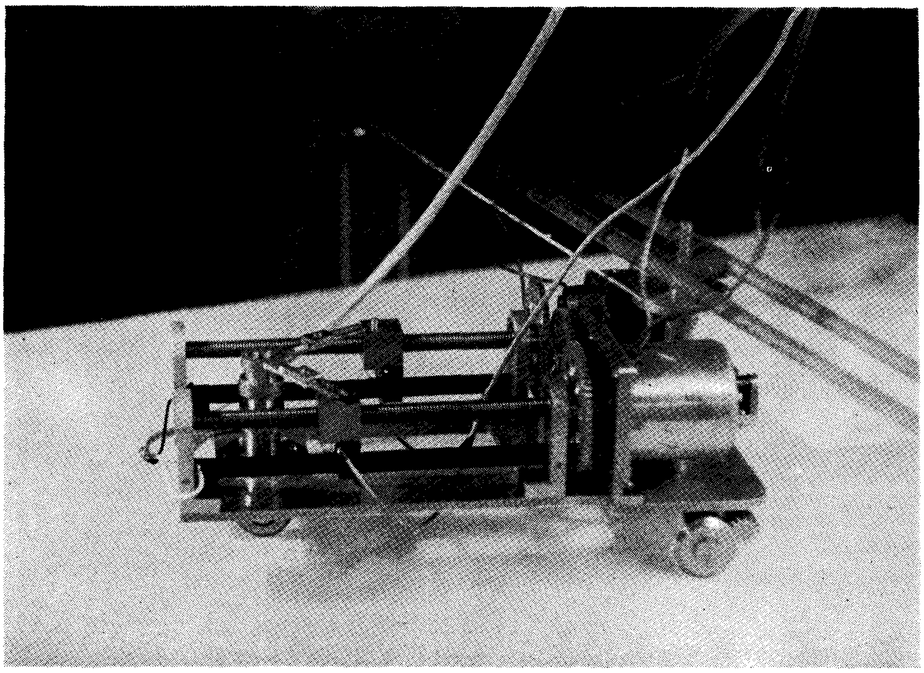

Photo. 9. Trolley plotting the path.

passes through the center of potentiometer *C*, the *L* type lever is kept away and the other end of the *L* type lever is pulled by the other contact mechanism of potentiometer. Thus the radius of curvature of trajectory can be given from the balancing of potentiometer. A pencil is fixed at the center of the back wheel shaft and pushed to the plotting board by spring.

The edge of the back wheels are made so sharp that slipping must be avoided during the motion.

(vi) Gantry mechanism

To make coincident the position and the rotating angle of the pair of probes with those of the trolley, the gantry is used as shown in Fig. 22. This gantry

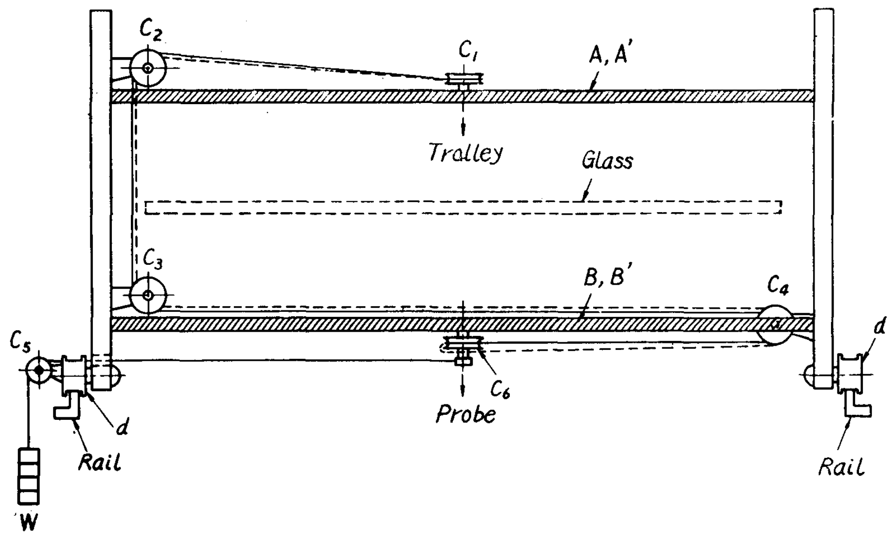

Fig. 22. Gantry mechanism connecting trolley and probes.

travels on the rails which fixed in parallel with the long side of electrolytic tank. There are two guide rails(AA' and BB') attached to the gantry. They are parallel with each other and exist in the same vertical plane to the electrolyte surface. The tracing trolley travels along the upper guide rail A A′, while the probes travel along the lower one B B′. Both probes and trolley are connected by the pulley mechanism (C_1 C_2 ⋯ C_6 shown in Fig. 22) for the purpose of the kinematical coincidence with each other.

5. Process of operation

The process of operation of this automatic electron trajectory tracer is as follows:

(1) Model electrodes are fixed near the center of electrolytic tank to avoid the effect of wall side on the true electric field.

(2) The model electrodes are represented on the plotting paper with care of the coincidence of their position with each other.

(3) After setting the trolley at the starting point, the oscillator circuit is closed.

(4) The circuits of V amplifier, null-amplifier, power supply, motor control unit and plate power supply of thyratron tube are closed except the circuit of ε_n amplifier. Then, the trolley is faced to the direction normal to the radiation surface of electron.

(5) The circuit of ε_n amplifier is closed.

(6) The driving motor is operated.

The trajectory having an extremely small radius of curvature cannot be plotted, because the limit of radius of curvature depends on the distance H (Fig. 14) and the length of the shaft of back wheels.

6. Results of experiment

It is necessary to check the accuracy of this automatic electron trajectory plotter before using it for practical purposes. Now let us consider the parallel electrodes of a cm interval when voltage V is applied between them as shown in Fig. 23, where V_0 is the accelerlating voltage of electron, and v_0 is the velocity of electron. Then the following relation is satisfied for v_0 and V_0:

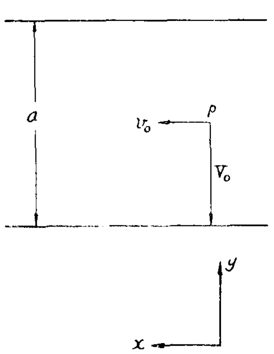

Fig. 23. Explanatory diagram of electron path between parallel electrodes.

$$\frac{1}{2} m v_0^2 = e V_0 . \qquad (24)$$

Assume that V_0 coinsides with as the potential at point P. Then the following relation holds if we take into account the X, Y coordinate on the plane where the electron projects:

$$y = \frac{1}{2} \frac{e}{m} \frac{V}{a} t^2 , \qquad (25)$$

where t is the time. Since the velocity to X-direction is constant,

$$y = \frac{1}{2}\frac{e}{m}\frac{V}{a}\left(\frac{x}{v}\right)^2 \tag{26}$$

when $t = x/v_0$.

From (25) and (26)

$$y = \frac{1}{4a}\left(\frac{V}{V_2}\right)x^2. \tag{27}$$

The results of mathematical computation are represented by the cross mark in Fig. 24. The solid lines show the results of actual experiment. In comparison with mathematical calculation, this automatic electron trajectory plotter seems to give sufficient accuracy for practical use.

7. Accuracy of the plotter

The accuracy of this equipment can be considered as follows:

(i) Polarization of liquid

If the electrode voltage is of low frequency, a homogeneous electric field cannot be formed by the occurrence of the polarization of liquid.

In this equipment, the electrede voltage of 1500 cycle is supplied because the

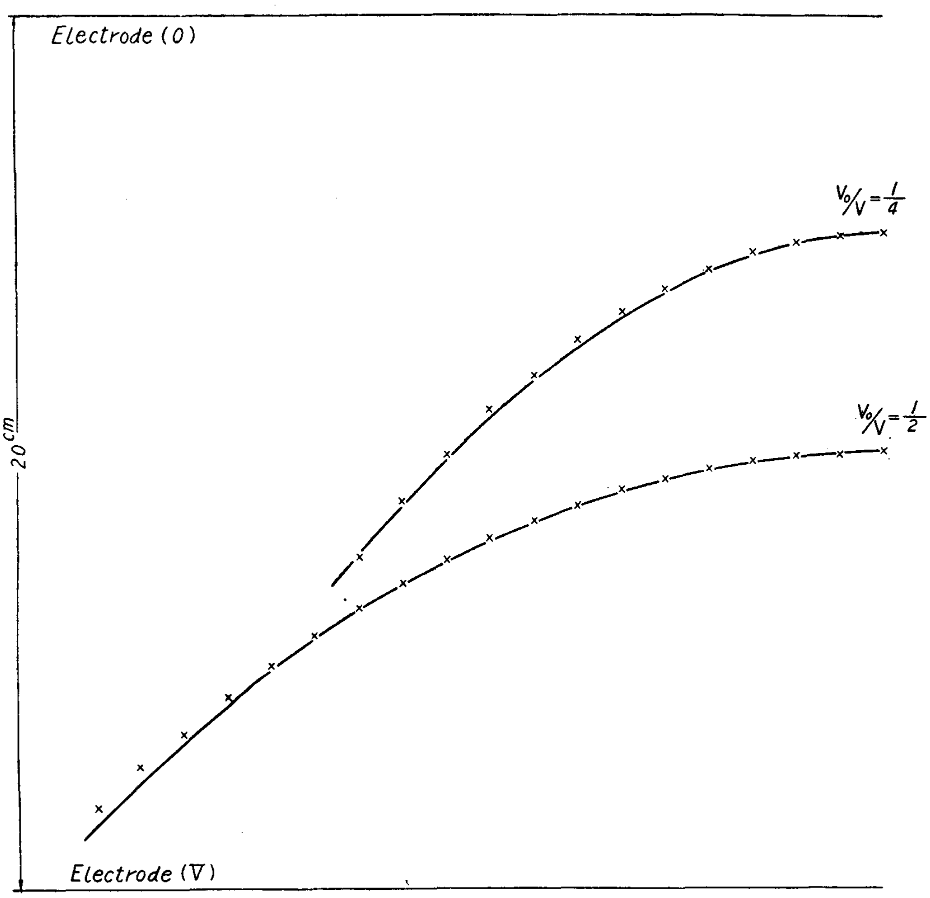

Fig. 24. Traced curve comparing with mathematical calculation (marked by cross).

polarizateon becomes to the smaller, the higher the voltage than 1000 cycly in frequency is applied. The polarization is particularly remarkable near the probes, rather than near the model electrodes.

(ii) Perturbation of the field by insertion of the probe

Since the perturbation of the electric field occurs, by submerging the probe, it is necessary to ristrict the probe current as small as possible. The depth of probe was about 3 mm in this equipment. If it is too short, the input signals from these probes are disturbed by high noise level, as described in chapter II.

(iii) Mechanical error

The mechanical errors of this equipment are induced from the deviation of the coincidence of positions of the probe and the pencil, and the inclination of the glass surface to the horizonal plane. The coincidence of probe and pencil was measured by the cathetometer. The inclination of the surface of the glass plate was measured by the level meter. The strip which was used to connect the probe and the pencil in the pulley system must not show shrinkage by moisture. By shrinkage of this strip, a displacement of 2 mm was found after two months.

Considering the electric circuit, the unbalanced current of the input transformer of potential amplifier and the stabilities of V and ε_n amplifiers are the essential factors in deciding the accuracy.

V. Conclusion

1. Electrolytic tank analogue

It has been well known that the electrolytic tank analogue is a convenient tool for measuring potential, potential gradient and electron trajectory in electrostatic field, and also for analyzing analogically another phenomena such as can be represented by Laplacian equation, for example, thermal conducting field, stress of elastic material, hydraulic fluid and so on. It is expected that the analogue can be extended to measuring the field where Laplacian equation cannot hold. For the purpose of practical use, the accuracy of electrolytic tank analogue was examined.

The factors to be considered as causing errors in measurement are as follows:

(1) Perturbation of the electrostatic field by insertion of a probe tip.

(2) Polarization of liquid near the electrodes.

(3) The effect of surface tension of the liquid.

(4) Mechanical error.

Although (1), (2) and (3) are related with one another, (1) is usually affected by the submerged depth of the probe tip. The length of depth should be short enough to avoid the perturbation of the field caused by the current of probe circuit. On the other hand, as a short length of depth raises the difficulty which makes the input circuit impedance of null-amplifier too high, the length should be determined in compromise of these problems.

With respect to (2), it was found that the tap water is good electrolyte for the present purpose. And the electrode of graphite-coated sheet is useful in avoiding polarization. It is, however, a complicated ploblem to choose a good electrolyte and a electrode for the design of electrolytic tank analogue, because the best choice also depends upon the object to be measured. The details of the experimental works of accuracy will be reported later.

(3) is a serious problem in the case of two adjacent probes prepared in the measurement of potential gradient and electron trajectory. To avoid this effect a better method will be required.

A description of (4) was given in the section of practical plotters.

2. Application of servomechanism

Chapters III and IV were devoted to the detailed description of the application of automatic control to the electrolytic tank analogue.

Since the electrode of the automatic equipotential line plotter is supplied with the voltage of 50 cycle in frequency, the errors caused by polarization is considered to be rather large. A skilful elimination of the hunting phenomenon will be expected with due regard to the mechanical friction. The application of this equipment may be extended widely to the field where the accuracy is necessary.

In the automatic electron trajectory tracer, the detection of potential gradient with high accuracy is difficult. On the other hand, many problems remain to be solved as to the mechanical construction. The equipments which were constructed by the authors are considered to be useful and convinient in many fields in the present condition. The field of application can be found not only in electrostatic or electronic engineering but also in mechanical and physical problems.

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Appendix

A. C. potentiometer for the measurement of phase difference and amplitude

It is often necessary to measure the amplitude and the phase difference between two A.C. voltages in the field of electronic engineering, especially in the design of servomechanism. For example, in the design of the automatic electron trajectory plotter, the motor control unit shown in Fig. 20 requires a comparison of the reference voltage with the input voltage to see wheather they are just in phase or 180° out of phase. For the purpose of such a measurement, the present authors made an A. C. potentiometer, which can be used to measure the amplitude and the phase shift of A. C. voltage, comparing with the other A. C. voltage of the same frequency. This instrument is easy for operation and simple in mechanism. It can also be used in measuring the impedance of any element in a simple manner, for example, the polarization of liquid.

It is well known that sinusoidal A. C. voltage can generally be represented as follows:

$$A \sin (\omega t + \phi),$$

where A, ω, t represent amplitude, angular velocity and time respectively. This term can also be represented by

$$A e^{j(\omega t+\phi)},$$

where ϕ means phase shift. Thus, when ω is known, the voltage and the phase shift can be evaluated by the knowlege of A and ϕ. If it is assumed,

$$A e^{j(\omega t+\phi)} = A \cos (\omega t+\phi) + j A \sin (\omega t+\phi)$$
$$= a + j b,$$

ϕ and A can be evaluated by the knowlege of a and b from the chart. For the purpose of measuring a and b, it is convenient if these unknown voltages are compared with the reference voltage a_0 and b_0 which are 90° out of phase with each other, having the same frequency. The A. C. potentiometer is shown in Fig. 25. Input voltage of phase splitter as a reference voltage can usually be taken from the system to be tested.

From the output of the phase splitter, a_o is fed into the input of quadrature generator which produces the voltage 90° out of phase with a_0. This output a_0 can be measured by the detector voltmeter. (The voltmeter is read 10 V in fullscale, and the ratio is indicated by the scale of variabe resistance VR_1 shown in Fig. 26). The same operation is repeated for reading b_0 (the ratio is read by the scale of VR_3 shown in Fig. 25). After switching the voltmeter circuit into null-detector circuit, a_0, b_0 and an unknown voltage are added together. Thus, the value a is determined by adjusting the variable resistance VR_2 until the detector indicates the minimun value (the ratio a/a_0 is indicated in VR_2). Repeating the same

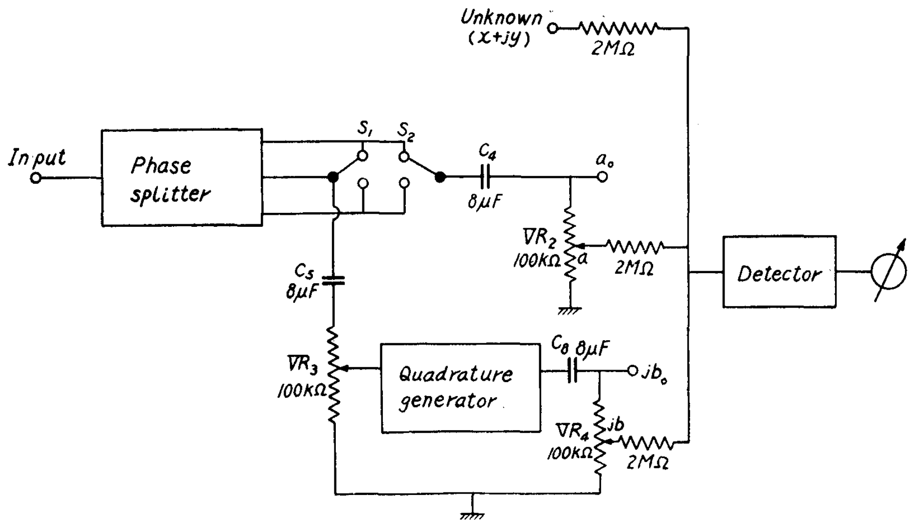

Fig. 25. Schematic diagram of A. C. potentiometer.

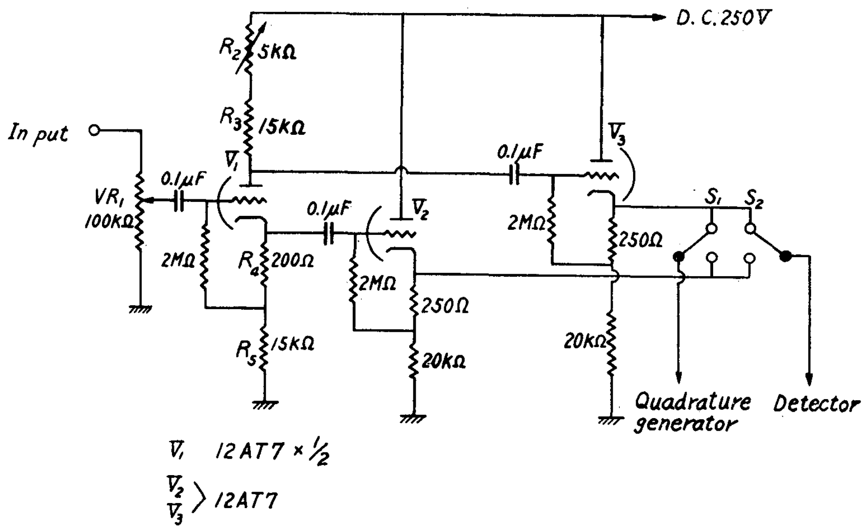

Fig. 26. Circuit diagram of phase splitter.

process, b can be indicated by adjusting variable resistance VR_4. The phase splitter is operated for the measurement of a and b in the overall domain of phase shift 360°. Therefore, if the zero value cannot be recognized in the detector voltmeter, it can be achieved by repeating the process after switching S_1 and S_2 adequately. This apparatus can be used for the measurement of *A. C.* voltage and phase shift in the frequency range from 2 cycle to 5 KC. Although the accuracy of this apparatus is not so high (at most 1 per cent), the rapid measurement of *A. C.* voltage is a great advantage for practical use. Furthermore, this apparatus can be used to measure the *A. C.* voltage containing high harmonics.

The details of the circuit are as follows: The phase splitter is shown in Fig. 26. This is a usual phase reversing circuit. The equality of the pushpul output signal is adjusted by varying the resistance R_2 which causes the output signal equal at $R_2 + R_3 = R_4 + R_5$, when the output load is small. The voltages between the anode and the earth, and the cathode and the earth are out phase with each other, and the phase difference with the input voltage depends only upon the time constant of the input $C\ R$ circuit (cf. Fig. 26).

The quadrature generator which generates the output signal 90° out of phase with a_0 is shown in Fig. 27. For the purpose of phase shift, the mutual inductance is not available because the constant output cannot be found in a wide frequency range. Therefore, the negative feedback integrator circuit was used. The gain of first stage tube is designed as high as possible for minimizing the deviation of phase shift from 90°. This error is computed as 0.03 radian in every frequency. Phase error is also avoided by taking off the coupling condensers of first and second stage. The condensers of integrator circuit may be changed in 12 steps according to the measuring frequency.

The detector which measures the value a_0 and b_0 and detects the unknown

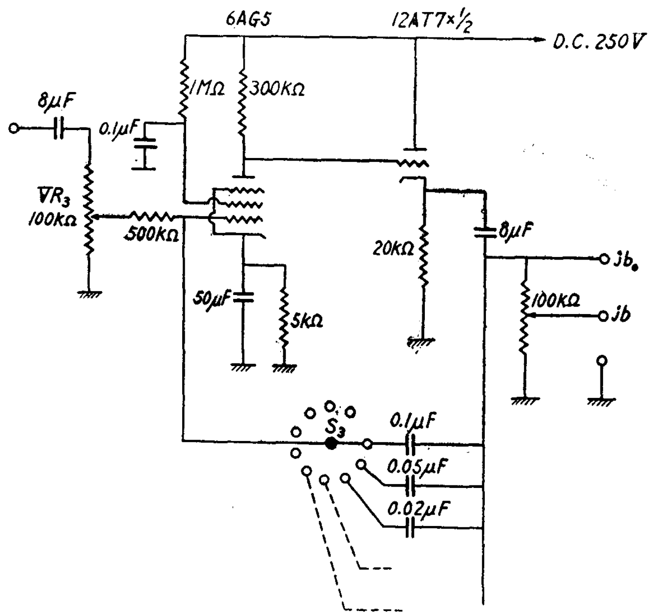

Fig. 27. Quadrature generator circuit.

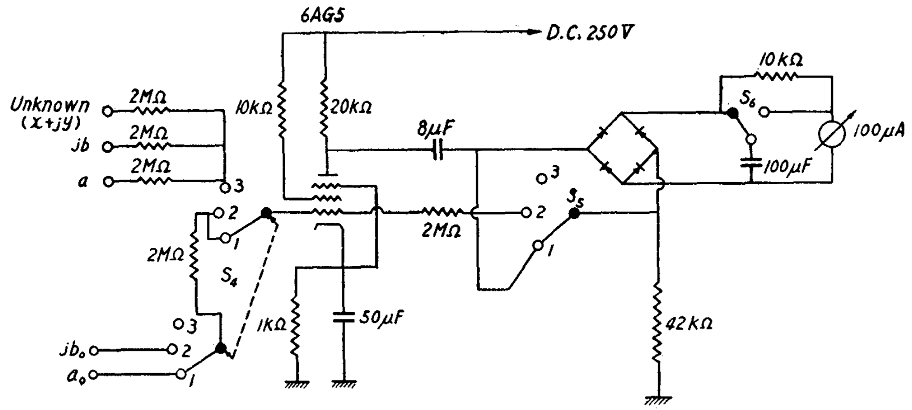

Fig. 28. Detector circuit.

voltage is shown in Fig. 28. This detector for a_0 and b_0 is also used as the null-detector by the switsh S_4. For measuring a_0 and b_0, the curcuit is used as the negative feedback voltmeter, by turning the switch S_5 to 2. For the detector of unknown voltage, on the other hand, switch S_5 is turned to 3, while to protect the voltmeter contact 1 is used. Switch S_6 is inserted to avoid the ripple in the measurement of very low frequency. The adjustment of full scale is done by varing the variable resistance of 42KΩ (cf. Fig.28). The coupling condenser of $8\mu F$ in the detector circuit is the oil condenser to avoid the *D. C.* leakage.

The voltage regulator is necessary to keep the suppling voltage constant for the all system.

Measurement of *A. C.* voltage and phase shift may thus be done by the following process:

(1) Turn the switch S_5 to voltmeter (Fig. 28),

(2) Turning the switch S_4 to 1 (Fig. 28), adjust a_0 to fullscale by VR_1 (Fig. 26).

(3) Adjust jb_0 in fullscale by turning S_4 to 2 (Fig. 28) while changing S_3 (Fig. 27) to adequate frequency range.

(4) Turning S_4 to 3, and S_5 to detector side (Fig. 28), make voltmeter minimum by adjusting VR_3 (Fig. 27), and make zero by VR_4 (Fig. 25).

(5) If the minimum cannot be available, repeat this process after turning the S_1, S_2 (Fig. 25) adequately

The term $a+jb$ can be transformed to the form $A\ e^{\phi}$ by the chart which is usually used. The amplitude and the phase difference of *A. C.* voltage can thus be compared with reference voltage by this system. Photo, 10 shows the overall

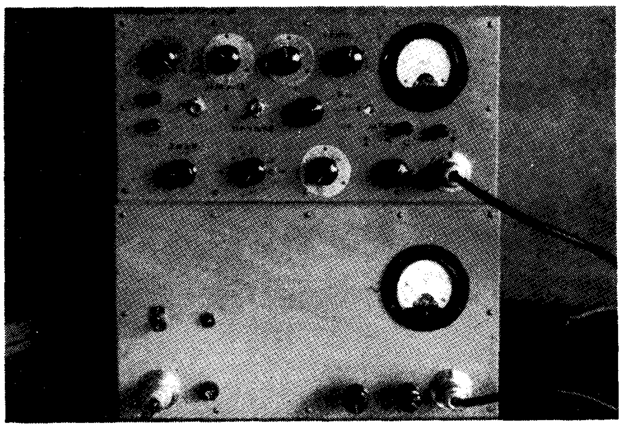

Photo. 10. Overall view of A.C. potentiometer.
Upper: A.C. potentiometer
Lower: D.C. voltage supply and voltage regulator.

view of *A. C.* potentiometer. This system has an advantage in simple mechanism and easy operation, although accuracy is not necessarily high. So, it may be considered to be useful for a rapid measurement of *A. C.* voltage or a comparison of two *A. C.* voltage, especially in the experiment on servomechanism, as described in the section of the motor control unit of automatic electron trajectory plotter. Furthermore, this apperatus may be useful for the measurement of the impedance of any element of the network rapidly.